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COLLEGE OF BUSINESS PEACE LEADERSHIP AND GOVERNANCE (CBPLG)

DISCRETE MATHEMATICS – CSC 201

END OF FIRST SEMESTER EXAMINATIONS

MAY/JUNE 2020

LECTURER: Mr. Weston Govere

DURATION: 3 HOURS

INSTRUCTIONS

Answer **ANY ONE** of the given questions. Each question carry **50** marks.

Show all working on your answer sheet.

The marks allocated to **each** question are shown at the end of the part question.

Question 1

i) Suppose that during the most recent fiscal year, the annual revenue of **Acme Computer** was 138 billion dollars and its net profit was 8 billion dollars, the annual revenue of **Nadir Software** was 87 billion dollars and its net profit was 5 billion dollars, and the annual revenue of **Quixote Media** was 111 billion dollars and its net profit was 13 billion dollars. Determine the truth value of each of these propositions for the most recent fiscal year.

(a) Quixote Media had the largest annual revenue.

(b) Nadir Software had the lowest net profit and Acme Computer had the largest annual revenue.

(c) Acme Computer had the largest net profit or Quixote Media had the largest net profit.

(d) If Quixote Media had the smallest net profit, then Acme Computer had the largest annual revenue.

(e) Nadir Software had the smallest net profit if and only

[5]

ii) Taking the long view on your education, you go to Econet Wireless Network Pvt Ltd and ask what you should do in university to be hired when you graduate. The personnel director replies that you will be hired only if you major in mathematics or computer science, get at least a B^+ average and take an accounting course. You do, in fact, become a mathematics major, get a B average and take an accounting course. Upon completion you return to Econet Wireless Network Pvt Ltd, make a formal application, and are turned down. Did the personnel director lie to you?

[4]

iii) A committee of three individuals decides issues for an organization. Each individual votes either yes or no for each proposal that arises. A proposal is passed if it receives at least two yes votes. Design a circuit that determines whether a proposal passes. **[4]**

iv) Sometimes light fixtures are controlled by more than one switch. Circuits need to be designed so that flipping any one of the switches for the fixture turns the light on when it is off and turns the light off when it is on. Design circuits that accomplish this when there are two switches and when there are three switches. **[4]**

v) Using Boolean algebra, show that $\left(p \wedge \left(\neg(\neg p \vee q)\right)\right) \vee (p \wedge q) \equiv p$ **[4]**

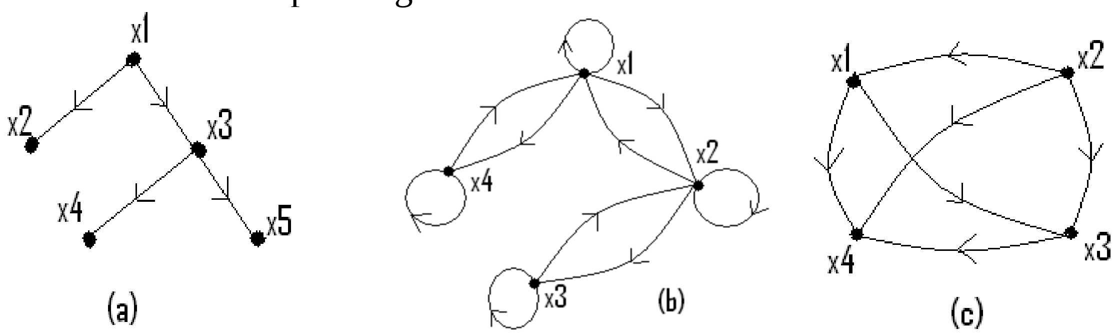
vi) Construct a combinatorial circuit using **inverters**, **OR gates**, and **AND gates** that produces the output $((\neg p \vee \neg r) \wedge \neg q) \vee (\neg p \wedge (q \vee r))$ from input bits $p, q,$ and r. **[4]**

vii) Determine whether the relation *R* on the set of all Web pages is reflexive, symmetric, antisymmetric, and/or transitive, where *(a, b)* ∈ *R* if and only if

(a) everyone who has visited Web page *a* has also visited Web page *b*.
(b) there are no common links found on both Web page *a* and Web page *b*.
(c) there is at least one common link on Web page *a* and Web page *b*.
(d) there is a Web page that includes links to both Web page *a* and Web page *b*.

[2, 2, 2, 2]

viii) Determine the properties of the relations given by graphs shown below and also write the corresponding relation matrices.



ix)

[9]

x)

TABLE 8 Flights.				
Airline	*Flight_number*	*Gate*	*Destination*	*Departure_time*
Nadir	122	34	Detroit	08:10
Acme	221	22	Denver	08:17
Acme	122	33	Anchorage	08:22
Acme	323	34	Honolulu	08:30
Nadir	199	13	Detroit	08:47
Acme	222	22	Denver	09:10
Nadir	322	34	Detroit	09:44

Using Table 8 above:

(a) List the 5-tuples in the relation. **[4]**

(b) Assuming that no new n-tuples are added, find a composite key with two fields containing the *Airline* field for the database. **[4]**

Question 2

i) Using the Euclidean algorithm find integers s and t satisfying that

$$\gcd(190, 34) = s(190) + t(34).$$ **[5]**

ii) Determine whether each of the functions $f(a) = a$ **div** d and $g(a) = a$ **mod** d, where d is a fixed positive integer, from the set of integers to the set of integers, is one-to-one, and determine whether each of these functions is onto.

[5]

iii) Find

(a) an inverse of 101 modulo 4620. **[3]**

(b) 7^{222} **mod** 11 **[3]**

iv) In this question suppose that Alice and Bob have these public keys and corresponding private keys: $(n_{Alice}, e_{Alice}) = (2867, 7) = (61 \cdot 47, 7)$, $d_{Alice} = 1183$ and $(n_{Bob}, e_{Bob}) = (3127, 21) = (59 \cdot 53, 21)$, $d_{Bob} = 1149$. First express your answers without carrying out the calculations. Then, using a computational aid, if available, perform the calculation to get the numerical answers.

a) Alice wants to send to all her friends, including Bob, the message "SELL EVERYTHING" so that he knows that she sent it. What should she send to her friends, assuming she signs the message using the RSA cryptosystem. **[5]**

b) Alice wants to send to Bob the message "BUY NOW" so that he knows that she sent it and so that only Bob can read it. What should she send to Bob, assuming she signs the message and then encrypts it using Bob's public key? **[5]**

v) Describe the steps that Alice and Bob follow when they use the Diffie-Hellman key exchange protocol to generate a shared key. (You may use some computational aid.) **[4]**

vi) Let f_1 and f_2 be functions from **R** to **R** such that $f_1(x) = x^2$ and $f_2(x) = x - x^2$. What are the functions $f_1 + f_2$ and $f_1 f_2$? **[2]**

vii) Find the domain and range of these functions.

(a) the function that assigns to each pair of positive integers the maximum of these two integers. **[2]**

(b) the function that assigns to each positive integer the number of the digits 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 that do not appear as decimal digits of the integer. **[2]**

viii) Find fog, gof where

$f(x) = 2x + 1$ and $g(x) = \frac{3}{x-1}$ $, x \neq 1$ **[2]**

ix) Let $f : \mathbf{Z} \rightarrow \mathbf{Z}$ be such that $f(x) = x + 1$. Is f invertible, and if it is, what is its inverse? **[3]**

x) If $f(x) = \frac{1}{x}$, show that $f(a) - f(b) = f\left(\frac{ab}{b-a}\right)$. **[2]**

xi) Show that if $ad - bc \neq 0$, then the function $f(x) = \frac{ax+b}{cx+d}$ is one-to-one and find its inverse. **[4]**

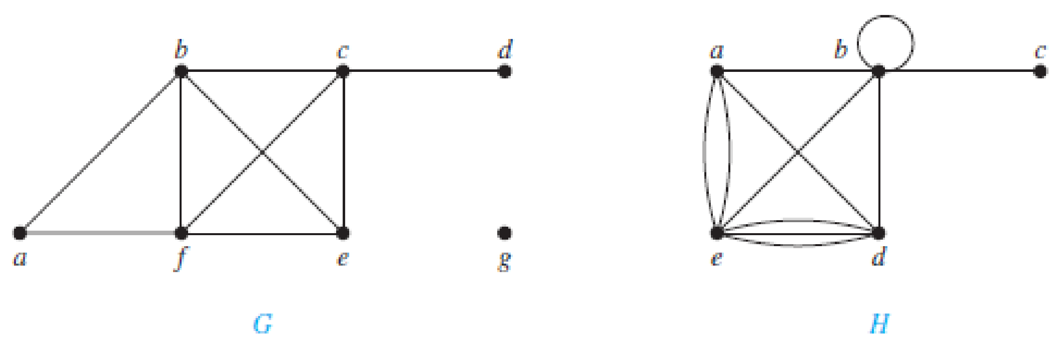
xii) Find the domain and range of the following function given by

$f(x) = \frac{\sqrt{(3x-5)(x+4)}}{x^3-16x}$. **[3]**

Question 3

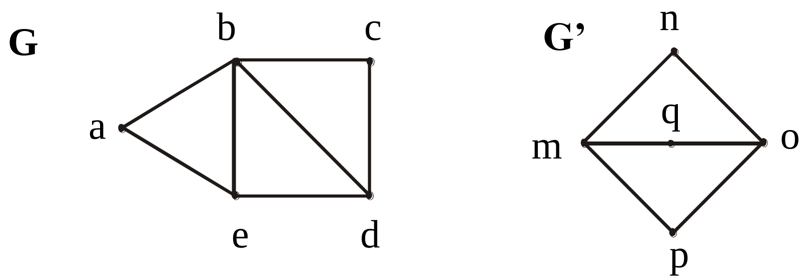
i) Prove that in a graph, the number of vertices with an odd degree is even. **[4]**

ii) What are the degrees and what are the neighbourhoods of the vertices in the graphs *G* and *H* displayed below?



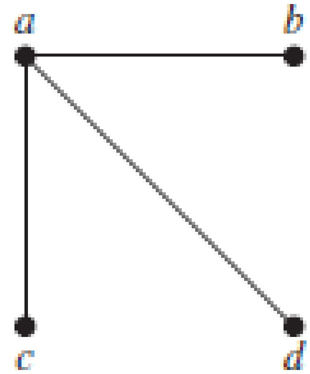
[5]

iii) Determine whether the graph G and G' given below are isomorphic.



[3]

iv) Draw five subgraphs of the following graph.



[4]

v) Suppose that a connected planner simple graph has 25 edges. If a plane drawing of this graph has 10 faces, how many vertices does this graph have?

[4]

vi) Mutare and Bulawayo are two of the cities competing for the National university games. (There are also many others). The organizers are narrowing the competition to the final 5 cities. There is a 20% chance that Mutare will be amongst the final 5. There is a 35% chance that Bulawayo will be amongst the final 5 and an 8% chance that both Mutare and Bulawayo will be amongst the final 5. What is the probability that Mutare or Bulawayo will be amongst the final 5. **[5]**

vii) What is the probability that the numbers 11, 4, 17, 39, and 23 are drawn in that order from a bin containing 50 balls labelled with the numbers 1, 2, . . . , 50 if

(a) the ball selected is not returned to the bin before the next ball is selected and

(b) the ball selected is returned to the bin before the next ball is selected?

[5, 4]

viii) Here you choose 6 numbers from the integers 1, 2, 3, …, 47, 48, 49. Six winning numbers are chosen together with a bonus number. How many choices for the 6 winning numbers. **[4]**

ix) A group of college students were asked about their TV watching habits. Of those surveyed, 28 students watch *The Walking Dead*, 19 watch *The Blacklist*, and 24 watch *Game of Thrones*. Additionally, 16 watch *The Walking Dead* and *The Blacklist*, 14 watch *The Walking Dead* and *Game of Thrones*, and 10 watch *The Blacklist* and *Game of Thrones*. There are 8 students who watch all three shows. How many students surveyed watched at least one of the shows? **[5]**

x) Suppose you are a game show contestant. You have a chance to win a large prize. You are asked to select one of three doors to open; the large prize is behind one of the three doors and the other two doors are losers. Once you select a door, the game show host, who knows what is behind each door, does the following. First, whether or not you selected the winning door, he opens one of the other two doors that he knows is a losing door (selecting at random if both are losing doors). Then he asks you whether you would like to switch doors. Which strategy should you use? Should you change doors or keep your original selection, or does it not matter? **[7]**

END OF EXAMINATION PAPER