**COLLEGE OF BUSINESS, PEACE, LEADERSHIP AND GOVERNANCE**

**NCSC102: ALGEBRA**

**END OF FIRST SEMESTER EXAMINATIONS**

**MAY/JUNE 2020**

**LECTURER: TARAMBAWAMWE P**

**DURATION: 48 Hours**

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## *INSTRUCTIONS*

You are required to answer ONE question only

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Credit will be awarded for logical, systematic and neat presentations

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**Question 1**

a. Let Matrix A = $\begin{pmatrix} 1 & -1 & 2 \\ 2 & 0 & -1 \\ -1 & 2 & -2 \end{pmatrix}$

i. If adj(A) = $\begin{pmatrix} 2 & -8 & a \\ 5 & c & 5 \\ b & -9 & 2 \end{pmatrix}$ find the values of a , b and c **[10 marks]**

b. Given that the augmented matrix $\left[\begin{array}{ccc|c} 1 & 3 & -1 & 8 \\ 0 & 3 & 1 & 11 \\ 0 & 0 & 4 & 8 \end{array}\right]$ represents a system of equations, give the solution to the system of equations using the inverse of a matrix. **[10 marks]**

c. i. Find the equation of the line joining points $P(2,1,-1)$ and $Q(0,3,1)$, in vector form **[ 5 marks]**

ii. Find the equation of the line $L$ passing through the points $A$ (3, 0, -1) and $B$ (5, -1, 4). In cartesian form. **[5 marks]**

**iii)** Determine whether or not the line $L$ above intersects the line with the equations

$$r = \begin{pmatrix} 5 \\ 1 \\ 11 \end{pmatrix} + \mu \begin{pmatrix} -4 \\ 1 \\ 3 \end{pmatrix}$$

**[ 10 marks]**

**d. Two lines A and B, have the following formulas:**

$$A = \begin{pmatrix} 4 \\ 0 \\ -2 \end{pmatrix} + \lambda \begin{pmatrix} -4 \\ 8 \\ -2 \end{pmatrix}$$

and

$$B = \begin{pmatrix} 6 \\ -10 \\ -10 \end{pmatrix} + \mu \begin{pmatrix} 2 \\ -6 \\ -2 \end{pmatrix}$$

**i.** determine whether these two lines intersect **[ 5 marks]**

**ii.** find the angle between them. **[ 5 marks]**

e) George and Hugo like to fly model airplanes. On one day George's plane takes off from level ground and shortly after that Hugo's plane takes off.

The position of George's plane *s*seconds after it takes off is given by $\mathbf{i}+2\mathbf{j}+s(5\mathbf{i}-2\mathbf{j}+6\mathbf{k})$ where the distances are in metres.

The position of Hugo's airplane *t* seconds after it takes off is given by $4\mathbf{i}-4\mathbf{j}+t(7\mathbf{i}-2\mathbf{j}+9\mathbf{k})$ where the distances are in metres.

Show that the paths of the planes are not perpendicular and at what point do the two airplanes collide. [**10 marks]**

**f. i.** Find the equation of the line joining points $P(2,1,-1)$ and $Q(0,3,1)$, in vector form**[ 5 marks]**

Q2

a ) Let Matrix A = $\begin{pmatrix} 1 & -1 & 2 \\ 2 & 0 & -1 \\ -1 & 2 & -2 \end{pmatrix}$

If adj(A) = $\begin{pmatrix} 2 & a & 1 \\ 5 & c & d \\ 4 & b & 2 \end{pmatrix}$ **[10 marks]**

find the values of a , b , c and d

b.

$$\begin{bmatrix} 1 & -1 & 0 \\ 0 & -m & 1 \\ \frac{a}{1+m} & -1 & 0 \end{bmatrix} \begin{bmatrix} Y \\ C_d \\ M \end{bmatrix} = \begin{bmatrix} I+X \\ 0 \\ 0 \end{bmatrix}$$

give the solution to the system of equations using Cramer's rule ie find Y and M.

**[10 marks]**

c) The points *A*, *B* and *C* have coordinates (2, –4, 6), (2, –3, –1) and (1, –2, 3).

i. Write down the position vectors of the points *A*, *B* and *C* relative to a fixed origin *O in i,j and k form,* . **[5 marks]**

ii. Write down the equation of the line *AB* in the form r = a + *t*b. **[5marks]**

iii. Find the equation of the plane passing through the three points(2, –4, 6), (2, –3, 1) and (1, –2, 3). **[5 marks]**

d)

i. The points *A*, *B* and *C* have coordinates (2, –4 6), (2, –3, –1) and (1, –2, 3).

The point *D* has coordinates (*a*, *b*, *c*) and lies on the line *AB* such that the line *CD* is perpendicular to the line *AB*.

Find the values of *a*, *b* and *c* and hence give the coordinates of the point *D*. [**15 marks]**

e) George and Hugo like to fly model airplanes. On one day George's plane takes off from level ground and shortly after that Hugo's plane takes off.

The position of George's plane *s*seconds after it takes off is given by i+2j+*s*(5i−2j+6k) where the distances are in metres.

The position of Hugo's airplane *t* seconds after it takes off is given by 4i−4j+*t(*7i−2j+9k) where the distances are in metres.

Show that the paths of the planes are not perpendicular and at what point do the two airplanes collide. [**10marks]**

**Question 3**

**i)** Given B = $\begin{pmatrix} 2 & -2 & 1 \\ 1 & 2 & -1 \\ 3+2x & 4+x & -2 \end{pmatrix}$ Find X if det(B) = -6 **[10 marks]**

**ii)** Find the inverse of matrix A if A = $\begin{pmatrix} 1 & 2 & 3 & 4 \\ 0 & -1 & 3 & -5 \\ 0 & 0 & -1 & -2 \\ 0 & 0 & 0 & 1 \end{pmatrix}$ **[10 marks]**

**iii.**

Relative to a fixed origin $O$, the point $A$ has position vector $(10\mathbf{i} + 2\mathbf{j} + 3\mathbf{k})$, and the point $B$ has position vector $(8\mathbf{i} + 3\mathbf{j} + 4\mathbf{k})$.

The line $l$ passes through the points $A$ and $B$.

(a) Find the vector $\overrightarrow{AB}$.

**(2)**

(b) Find a vector equation for the line $l$.

**(2)**

The point $C$ has position vector $(3\mathbf{i} + 12\mathbf{j} + 3\mathbf{k})$.

The point $P$ lies on $l$. Given that the vector $\overrightarrow{CP}$ is perpendicular to $l$,

(c) find the position vector of the point $P$.

**(6)**

Iii

a. Find the equation of the plane through $P(1,2,3)$, $Q(1,-1,1)$, and $R(3,2,1)$ **[5 marks]**

b. Find the equations of the planes parallel to $x - y + z = 2$ through

i. $P(0,2,0)$ **[5 marks]**

ii $R(3,2,1)$ **[5 marks]**

iv) George and Hugo like to fly model airplanes. On one day George's plane takes off from level ground and shortly after that Hugo's plane takes off. The position of George's plane *s*seconds after it takes off is given by i+2j+*s*(5i−2j+6k) where the distances are in metres. The position of Hugo's airplane *t* seconds after it takes off is given by 4i−4j+*t*(7i−2j+9k) where the distances are in metres.Show that the paths of the planes are not perpendicular and at what point do the two airplanes collide. [**10 marks]**

# End of Examination Paper