AFRICA UNIVERSITY
(A United Methodist-Related Institution)

"Investing in Africa's Future"

COLLEGE OF BUSINESS, PEACE, LEADERSHIP AND GOVERNANCE

QUANTITATIVE ANALYSIS I (MMS202)

FINAL EXAMINATION

MAY 2018

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3 HOURS

INSTRUCTIONS

Answer All questions.

Start each question on a new page in your answer booklet.

The marks allocated are shown at the end of each question.

Show all your workings.

Credit will be given for logical, systematic and neat presentations.

1. A study was conducted by Aventis pharmaceuticals to measure the adverse side effects of Allegra, a drug used for treatment of seasonal allergies. A sample of 679 allergy sufferers in the country X was given 60 mg of the drug twice a day. The patients were to report whether they experienced relief from their allergies as well as any adverse side effects (viral infection, nausea and drowsiness).
   a) What is the population of the study? **[2 marks]**
   b) What is the sample of the study? **[2 marks]**
   c) What are the characteristics of interest of each element in the population? **[4 marks]**

2. A telecommunication company frequently receives reports on problem with the telephone lines. The following data shows the time (in minutes) used to solve these problems from the customers' lines.

8.9	8.4	7.3	4.0	6.7	4.5	6.3
9.7	3.3	1.1	8.4	4.3	7.6	3.5

   Calculate
   a) Mean **[2 marks]**
   b) Median **[3 marks]**
   c) Mode **[2 marks]**
   d) Standard deviation **[3 marks]**
   e) What is the distribution of the data? **[2 marks]**

3. There are 100 students enrolled in the Department of Sciences. The courses offered by the Department are Mathematics (M), Physics (F) and Chemistry (K).

   10 students enrolled all courses.
   25 students enrolled in Mathematics and Physics courses.
   20 students enrolled in Physics and Chemistry courses.
   28 students enrolled in Mathematics and Chemistry courses.
   60 students enrolled in Mathematics course.
   50 students enrolled in Physics course.
   53 students enrolled in Chemistry course.

   (a) Construct a Venn diagram. **[3 marks]**
   (b) How many students do not enrolled in either Mathematics course or Physics course? **[3 marks]**
   (c) Based on (a), if a student is randomly selected, what is the probability that the student is:
      i. enrolled in only one course? **[2 marks]**
      ii. enrolled in Physics and Chemistry courses but not enrolled in Mathematics course. **[2 marks]**

4. A researcher wants to know whether running is related to longevity. From a sample of recently deceased runners, the average number of kilometres (estimated per day for their last five years) is paired with the number of years that they lived.

Subject	Distance (km)	Years Lived
1	25	63
2	35	68
3	10	72
4	40	62
5	85	65
6	75	46
7	60	51
8	45	60
9	50	55

a) Plot the scatterplot and interpret. **[3 marks]**
b) Calculate Pearson's correlation coefficient and interpret. **[4 marks]**
c) Fit the regression equation and estimate the number of years lived when the number of cigarettes is 100. **[4 marks]**

5. A research was conducted to know how many children below 12 years old have toy(s). The table shows the data.

Number of toy	Number of children
0	3
1	6
2	10
3	4
4	4
5	3

a) Construct and plot the probability distribution table of the children below 12 years old who have toys. **[5 marks]**
b) Calculate the standard deviation of the probability distribution table. **[5 marks]**
c) What is the probability of selecting a child with
   i. between 1 and 4 toys? **[2 marks]**
   ii. at most 3 toys? **[2 marks]**

6. Given the following data:

Weight (kg)	Frequency
5 – 7	3
8 – 10	11
11 – 13	12
14 – 16	8
17 -19	8
20 – 22	4

(a) Calculate the measures of central location. What is the distribution of the data? **[9 marks]**

(b) Calculate the coefficient of variation and interpret **[6 marks]**

7. A bulletin reported that children between the ages of 2 and 5 watch an average of 25 hours of television per week. Assume the variable is normally distributed and the standard deviation is 3 hours. If 20 children between the ages of 2 and 5 are randomly selected, find the probability that the mean of the number of hours they watch television is
   a) greater than 26.3 hours. **[2 marks]**
   b) less than 24 hours **[2 marks]**
   c) between 24 and 26.3 hours. **[2 marks]**

8. A researcher claims that the average wind speed in a certain city is 8km per hour. A sample of 32 days has an average wind speed of 8.2km per hour. The standard deviation of the population is 0.6km per hour.
   a) At $\alpha = 0.05$, is there enough evidence to reject the claim? Use the P-value method. **[6 marks]**
   b) Use a 95% confidence interval to verify your results in (a) **[5 marks]**
   c) Show that the sample mean is an unbiased estimator of the population mean **[3 marks]**

9. In a sample of 200 workers, 45% said that they missed work because of personal illness. Ten years ago in a sample of 200 workers, 35% said that they missed work because of personal illness.
   a) At $\alpha = 0.01$, is there a difference in the proportion of workers who missed work because of personal illness? **[6 marks]**
   b) Construct a 99% confidence interval for the difference between the proportion of workers who missed work due to illness **[4 marks]**

End of paper

ADDITIONAL INFORMATION

1. Sturge's Rule:

   Number of class, $C = 1 + 3.3\log n$

   Class width, $i > \frac{range}{C}$

2. Mean of grouped data $= \frac{\sum_{i=1}^{n} fx_i}{n}$

3. Mean of ungrouped data $= \frac{\sum_{i=1}^{n} x_i}{n}$

4. Mode $= L_{mo} + \left(\frac{\Delta_1}{\Delta_1 + \Delta_2}\right)i$

5 Median $= L_{me} + \left(\frac{\frac{n}{2} - F}{f_m}\right)i$

6. Standard deviation: $s = \sqrt{\frac{\sum_{i=1}^{n} fx_i^2 - \frac{\left(\sum_{n=1}^{n} fx_i\right)^2}{n}}{n-1}}$

7. Standard Deviation of ungrouped data: $s = \sqrt{\frac{\sum_{i=1}^{n} x_i^2 - \frac{\left(\sum_{i=1}^{n} x_i\right)^2}{n}}{n-1}}$

8. Coefficient of skewness: $S_k = \frac{3(mean - median)}{s} = \frac{mean - \text{mod}e}{s}$

9. Conditional probability: $P(A \setminus B) = \frac{P(A \cap B)}{P(A)}$

10. Binomial Distribution
    - $P(X = x) = nC_x p^x q^{n-x}$

11. Poisson Distribution
    - $P(X = x) = \frac{e^{-\lambda}\lambda^x}{x!}$

12. Hypothesis testing (single mean)
    - $Z = \frac{\bar{X} - \mu}{\frac{\sigma}{\sqrt{n}}}$
    - $t = \frac{\bar{X} - \mu}{\frac{s}{\sqrt{n}}}$, $df = n - 1$

13. Hypothesis testing (single proportion)
    - $Z = \frac{p - \pi}{\sqrt{\frac{\pi(1-\pi)}{n}}}$

14. Hypothesis testing (difference of two means)

- $Z = \frac{\bar{X}_1 - \bar{X}_2}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$
- $t = \frac{\bar{X}_1 - \bar{X}_2}{\sqrt{\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}}}$, df = smaller $(n_1 - 1;\ n_2 - 1)$
- $t = \frac{\bar{X}_1 - \bar{X}_2}{\sqrt{\frac{S_p^2}{n_1} + \frac{S_p^2}{n_2}}}$

  Where $S_p^2 = \frac{S_1^2(n_1) + S_2^2(n_2)}{n_1 + n_2 - 2}$, $df = n_1 + n_2 - 2$
- $t = \frac{\bar{D} - \mu_D}{\frac{S_D}{\sqrt{n}}}$, df = n – 1

15. Hypothesis testing (difference of two proportions)

- $Z = \frac{p_1 - p_2}{\sqrt{\bar{p}\bar{q}\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$

16. Confidence Interval (Single mean)

- $\bar{X} - Z_{\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}} \leq \mu \leq \bar{X} + Z_{\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}}$
- $\bar{X} - t_{\frac{\alpha}{2}} \frac{S}{\sqrt{n}} \leq \mu \leq \bar{X} + t_{\frac{\alpha}{2}} \frac{S}{\sqrt{n}}$

17. Confidence Interval (Difference of two means)

- $(\bar{X}_1 - \bar{X}_2) - Z_{\frac{\alpha}{2}} \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}} \leq (\mu_1 - \mu_2) \leq (\bar{X}_1 - \bar{X}_2) + Z_{\frac{\alpha}{2}} \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$
- $(\bar{X}_1 - \bar{X}_2) - t_{\frac{\alpha}{2}} \sqrt{\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}} \leq (\mu_1 - \mu_2) \leq (\bar{X}_1 - \bar{X}_2) + t_{\frac{\alpha}{2}} \sqrt{\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}}$

  df = smaller $(n_1 - 1;\ n_2 - 1)$
- $(\bar{X}_1 - \bar{X}_2) - t_{\frac{\alpha}{2}} \sqrt{\frac{S_p^2}{n_1} + \frac{S_p^2}{n_2}} \leq (\mu_1 - \mu_2) \leq (\bar{X}_1 - \bar{X}_2) + t_{\frac{\alpha}{2}} \sqrt{\frac{S_p^2}{n_1} + \frac{S_p^2}{n_2}}$

  Where $S_p^2 = \frac{S_1^2(n_1) + S_2^2(n_2)}{n_1 + n_2 - 2}$, $df = n_1 + n_2 - 2$

18. Confidence Interval (Single proportion)

- $p - Z_{\frac{\alpha}{2}} \sqrt{\frac{pq}{n}} \leq \pi \leq p + Z_{\frac{\alpha}{2}} \sqrt{\frac{pq}{n}}$

19. Confidence Interval (Difference of two proportions)

- $(p_1 - p_2) - Z_{\frac{\alpha}{2}} \sqrt{\frac{p_1 q_1}{n_1} + \frac{p_2 q_2}{n_2}} \leq (\pi_1 - \mu_2) \leq (p_1 - p_2) + Z_{\frac{\alpha}{2}} \sqrt{\frac{p_1 q_1}{n_1} + \frac{p_2 q_2}{n_2}}$

20. $Z = \frac{X - \mu}{\sigma}$

21. Weighted Mean: $\bar{X}_w = \frac{\sum xw}{\sum w}$

22. $P(B_i / C) = \dfrac{P(C / B_i)P(B_i)}{\sum_{i=1}^{n} P(C / B_i)P(B_i)}$,

23. $y = \beta_0 + \beta_1 x + e$,

$$\beta_1 = \frac{n\sum xy - \sum x \sum y}{n\sum y^2 - (\sum y)^2}$$

$$\beta_0 = \bar{y} + \beta_1 \bar{x}$$

$$r = \frac{n\sum xy - \sum x \sum y}{\sqrt{\{(n\sum x^2 - (\sum x)^2)(n\sum y^2 - (\sum y)^2)\}}}$$

24. Let f_0 and f_e be the observed and expected frequencies respectively:

$$\chi^2 = \sum \frac{(f_o - f_e)^2}{f_e}$$

Extract From Random Number Table

22	17	68	65	84	68	95	23	92	35	87	02	22
57	55	61	09	43	95	06	58	24	82	03	47	10
27	53	96	23	71	50	54	36	23	54	31	04	82
98	04	14	12	15	09	26	78	25	47	47		

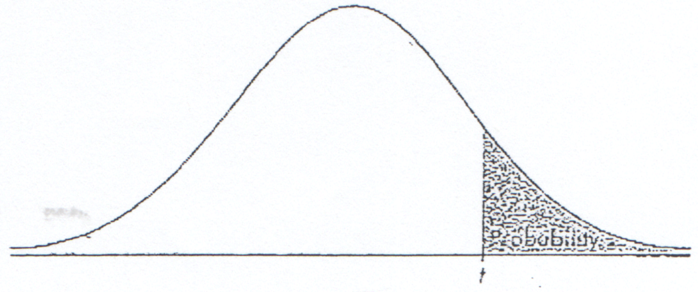


TABLE B: *t*-DISTRIBUTION CRITICAL VALUES

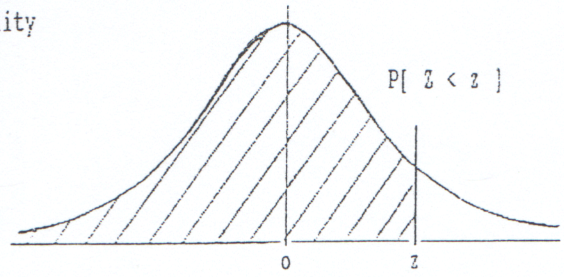
	Tail probability p											
df	.25	.20	.15	.10	.05	.025	.02	.01	.005	.0025	.001	.0005
1	1.000	1.376	1.963	3.078	6.314	12.71	15.89	31.82	63.66	127.3	318.3	636.6
2	.816	1.061	1.386	1.886	2.920	4.303	4.849	6.965	9.925	14.09	22.33	31.60
3	.765	.978	1.250	1.638	2.353	3.182	3.482	4.541	5.841	7.453	10.21	12.92
4	.741	.941	1.190	1.533	2.132	2.776	2.999	3.747	4.604	5.598	7.173	8.610
5	.727	.920	1.156	1.476	2.015	2.571	2.757	3.365	4.032	4.773	5.893	6.869
6	.718	.906	1.134	1.440	1.943	2.447	2.612	3.143	3.707	4.317	5.208	5.959
7	.711	.896	1.119	1.415	1.895	2.365	2.517	2.998	3.499	4.029	4.785	5.408
8	.706	.889	1.108	1.397	1.860	2.306	2.449	2.896	3.355	3.833	4.501	5.041
9	.703	.883	1.100	1.383	1.833	2.262	2.398	2.821	3.250	3.690	4.297	4.781
10	.700	.879	1.093	1.372	1.812	2.228	2.359	2.764	3.169	3.581	4.144	4.587
11	.697	.876	1.088	1.363	1.796	2.201	2.328	2.718	3.106	3.497	4.025	4.437
12	.695	.873	1.083	1.356	1.782	2.179	2.303	2.681	3.055	3.428	3.930	4.318
13	.694	.870	1.079	1.350	1.771	2.160	2.282	2.650	3.012	3.372	3.852	4.221
14	.692	.868	1.076	1.345	1.761	2.145	2.264	2.624	2.977	3.326	3.787	4.140
15	.691	.866	1.074	1.341	1.753	2.131	2.249	2.602	2.947	3.286	3.733	4.073
16	.690	.865	1.071	1.337	1.746	2.120	2.235	2.583	2.921	3.252	3.686	4.015
17	.689	.863	1.069	1.333	1.740	2.110	2.224	2.567	2.898	3.222	3.646	3.965
18	.688	.862	1.067	1.330	1.734	2.101	2.214	2.552	2.878	3.197	3.611	3.922
19	.688	.861	1.066	1.328	1.729	2.093	2.205	2.539	2.861	3.174	3.579	3.883
20	.687	.860	1.064	1.325	1.725	2.086	2.197	2.528	2.845	3.153	3.552	3.850
21	.686	.859	1.063	1.323	1.721	2.080	2.189	2.518	2.831	3.135	3.527	3.819
22	.686	.858	1.061	1.321	1.717	2.074	2.183	2.508	2.819	3.119	3.505	3.792
23	.685	.858	1.060	1.319	1.714	2.069	2.177	2.500	2.807	3.104	3.485	3.768
24	.685	.857	1.059	1.318	1.711	2.064	2.172	2.492	2.797	3.091	3.467	3.745
25	.684	.856	1.058	1.316	1.708	2.060	2.167	2.485	2.787	3.078	3.450	3.725
26	.684	.856	1.058	1.315	1.706	2.056	2.162	2.479	2.779	3.067	3.435	3.707
27	.684	.855	1.057	1.314	1.703	2.052	2.158	2.473	2.771	3.057	3.421	3.690
28	.683	.855	1.056	1.313	1.701	2.048	2.154	2.467	2.763	3.047	3.408	3.674
29	.683	.854	1.055	1.311	1.699	2.045	2.150	2.462	2.756	3.038	3.396	3.659
30	.683	.854	1.055	1.310	1.697	2.042	2.147	2.457	2.750	3.030	3.385	3.646
40	.681	.851	1.050	1.303	1.684	2.021	2.123	2.423	2.704	2.971	3.307	3.551
50	.679	.849	1.047	1.299	1.676	2.009	2.109	2.403	2.678	2.937	3.261	3.496
60	.679	.848	1.045	1.296	1.671	2.000	2.099	2.390	2.660	2.915	3.232	3.460
80	.678	.846	1.043	1.292	1.664	1.990	2.088	2.374	2.639	2.887	3.195	3.416
100	.677	.845	1.042	1.290	1.660	1.984	2.081	2.364	2.626	2.871	3.174	3.390
1000	.675	.842	1.037	1.282	1.646	1.962	2.056	2.330	2.581	2.813	3.098	3.300
∞	.674	.841	1.036	1.282	1.645	1.960	2.054	2.326	2.576	2.807	3.091	3.291
	50%	60%	70%	80%	90%	95%	96%	98%	99%	99.5%	99.8%	99.9%
	Confidence level C											

STANDARD STATISTICAL TABLES

1. Areas under the Normal Distribution

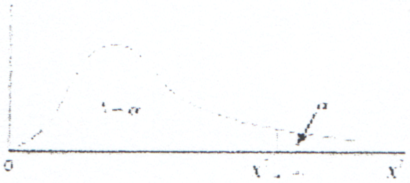
The table gives the cumulative probability up to the standardised normal value z
i.e.

$$P[\, Z < z \,] = \int_{-\infty}^{z} \frac{1}{\sqrt{2\pi}} \exp(-\tfrac{1}{2}Z^2)\, dZ$$



z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5159	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7854
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8804	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9773	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9865	0.9868	0.9871	0.9874	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9924	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9980	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986

z	3.00	3.10	3.20	3.30	3.40	3.50	3.60	3.70	3.80	3.90
P	0.9986	0.9990	0.9993	0.9995	0.9997	0.9998	0.9998	0.9999	0.9999	1.0000



Degrees of Freedom	.995	.99	.975	.95	.90	.75	.25	.10	.05	.025	.01	.005
						Upper Tail Areas (α)						
1			0.001	0.004	0.016	0.102	1.323	2.706	3.841	5.024	6.635	7.879
2	0.010	0.020	0.051	0.103	0.211	0.575	2.773	4.605	5.991	7.378	9.210	10.597
3	0.072	0.115	0.216	0.352	0.584	1.213	4.108	6.251	7.815	9.348	11.345	12.838
4	0.207	0.297	0.484	0.711	1.064	1.923	5.385	7.779	9.488	11.143	13.277	14.860
5	0.412	0.554	0.831	1.145	1.610	2.675	6.626	9.236	11.071	12.833	15.086	16.750
6	0.676	0.872	1.237	1.635	2.204	3.455	7.841	10.645	12.592	14.449	16.812	18.548
7	0.989	1.239	1.690	2.167	2.833	4.255	9.037	12.017	14.067	16.013	18.475	20.278
8	1.344	1.646	2.180	2.733	3.490	5.071	10.219	13.362	15.507	17.535	20.090	21.955
9	1.735	2.088	2.700	3.325	4.168	5.899	11.389	14.684	16.919	19.023	21.666	23.589
10	2.156	2.558	3.247	3.940	4.865	6.737	12.549	15.987	18.307	20.483	23.209	25.188
11	2.603	3.053	3.816	4.575	5.578	7.584	13.701	17.275	19.675	21.920	24.725	26.757
12	3.074	3.571	4.404	5.226	6.304	8.438	14.845	18.549	21.026	23.337	26.217	28.299
13	3.565	4.107	5.009	5.892	7.042	9.299	15.984	19.812	22.362	24.736	27.688	29.819
14	4.075	4.660	5.629	6.571	7.790	10.165	17.117	21.064	23.685	26.119	29.141	31.319
15	4.601	5.229	6.262	7.261	8.547	11.037	18.245	22.307	24.996	27.488	30.578	32.801
16	5.142	5.812	6.908	7.962	9.312	11.912	19.369	23.542	26.296	28.845	32.000	34.267
17	5.697	6.408	7.564	8.672	10.085	12.792	20.489	24.769	27.587	30.191	33.409	35.718
18	6.265	7.015	8.231	9.390	10.865	13.675	21.605	25.989	28.869	31.526	34.805	37.156
19	6.844	7.633	8.907	10.117	11.651	14.562	22.718	27.204	30.144	32.852	36.191	38.582
20	7.434	8.260	9.591	10.851	12.443	15.452	23.828	28.412	31.410	34.170	37.566	39.997
21	8.034	8.897	10.283	11.591	13.240	16.344	24.935	29.615	32.671	35.479	38.932	41.401
22	8.643	9.542	10.982	12.338	14.042	17.240	26.039	30.813	33.924	36.781	40.289	42.796
23	9.260	10.196	11.689	13.091	14.848	18.137	27.141	32.007	35.172	38.076	41.638	44.181
24	9.886	10.856	12.401	13.848	15.659	19.037	28.241	33.196	36.415	39.364	42.980	45.559
25	10.520	11.524	13.120	14.611	16.473	19.939	29.339	34.382	37.652	40.646	44.314	46.928
26	11.160	12.198	13.844	15.379	17.292	20.843	30.435	35.563	38.885	41.923	45.642	48.290
27	11.808	12.879	14.573	16.151	18.114	21.749	31.528	36.741	40.113	43.194	46.963	49.645
28	12.461	13.565	15.308	16.928	18.939	22.657	32.620	37.916	41.337	44.461	48.278	50.993
29	13.121	14.257	16.047	17.708	19.768	23.567	33.711	39.087	42.557	45.722	49.588	52.336
30	13.787	14.954	16.791	18.493	20.599	24.478	34.800	40.256	43.773	46.979	50.892	53.672