AFRICA UNIVERSITY
A United Methodist-Related Institution
"Investing in Africa's Future"

COLLEGE OF BUSINESS, PEACE, LEADERSHIP & GOVERNANCE

COURSE CODE AND TITLE: MEC 203-Mathematics for Economists

END OF FIRST SEMESTER EXAMINATION

NOVEMBER 2017

LECTURER: Mr L. NGENDAKUMANA

DURATION: 3 HOURS

INSTRUCTIONS

Answer **any five (5) questions**
Total possible mark is 100

Start **each** question on a new page in your answer booklet.

The marks allocated to **each** question are shown at the end of the section.

Show all your workings.

Credit will be awarded for logical, systematic and neat presentations.

Question 1

(i) Use Cramer's Rule to find Y and C when

$Y = C+Io+Go$ $\quad\quad\quad$ $C= a +bY$

Where Y is the national product and C is a private consumption. The symbols Io (private investment), Go (government consumption and investment), a and b all represent constants, with $b< 1$.

a. Use Cramer's rule and another simple method to find Y and C. **[10]**

b. Define and explain parameters a and b **[2]**

(ii) Let Y_t , I_t and S_t denote the national product, total investment and saving respectively, in Zimbabwe at time t. Suppose that savings are proportional to national income, and investment is proportional to the change in income:

$$S_t = \alpha Y_t \quad (1) \qquad I_t = \beta(Y_t - Y_{t-1}) \quad (2) \qquad S_t = I_t \quad (3)$$

$\alpha = 0.4$ and $\beta = 0.8$

(i) Deduce the difference equation for Y_t, **[4]**

(ii) Solve it for $Y_o= 4000$. **[4]**

Question 2

(i) Define both types of vectors and explain how dots products are computed. Use three concrete numerical examples to support your arguments **[6]**

(ii) Find the inverse of the following two matrices in (a) and (b):

(a)

$$\begin{pmatrix} 1 & 1 & -3 \\ 2 & 1 & -3 \\ 2 & 2 & 1 \end{pmatrix}$$

[4]

(b)

Let $P = \frac{1}{2}\begin{pmatrix} -1 & -\sqrt{3} \\ \sqrt{3} & -1 \end{pmatrix}$

[5]

Show that $P^3 = I$. Use this to find P^{-1}

(c) When are the two matrices below equal?

$$\begin{pmatrix} 3 & e-1 \\ 2e & f \end{pmatrix} = \begin{pmatrix} e & 2g \\ f+1 & i+e \end{pmatrix}$$

[5]

(d) An n x n matrix P is said to be orthogonal if P'P=In. For $\lambda = 1/\sqrt{2}$ show that the matrix below is orthogonal:

$$P = \begin{pmatrix} \lambda & 0 & \lambda \\ \lambda & 0 & -\lambda \\ 0 & 1 & 0 \end{pmatrix}$$

[4]

Question 3

A firm uses inputs K and L of capital and labor, respectively to produce a single output Q according to the production function $Q = F(K, L) = K^{\frac{1}{2}} L^{\frac{1}{4}}$. The prices of capital and labor are r and w, respectively.

a. Find the cost minimizing inputs of K and L, and also the minimum cost C, as functions of r, w and Q. denote the cost minimizing values by K*, L* and C*

[10]

b. Verify that $K^* = \frac{\partial C^*}{\partial r}, L^* = \frac{\partial C^*}{\partial w}, \lambda = \frac{\partial C^*}{\partial Q}, \frac{\partial K^*}{\partial w} = \frac{\partial L^*}{\partial r}$

where λ denotes the Lagrange multiplier. **[8]**

c. Use your result in (b) to provide an economic interpretation of the Lagrange multiplier **[2]**

Question 4

An individual has m dollars to spend on two commodities X and Y. The prices per unit of X and Y are p and q respectively. The utility enjoyed by consuming X and Y units is given by:

$$u(x, y) = xy$$

a. Formulate the maximization problem and write down the Lagrange function. **[2]**

b. Find the optimal values of x and y that solve the maximization problem. **[9]**

c. Find the Lagrange multiplier λ for this problem **[3]**

d. Let $u^*(m) = x(m)y(m)$ be the value function. Verify that $\dfrac{du^*(m)}{dm} = \lambda(m)$ **[3]**

e. What would be the additional utility if m increases by \$1? **[3]**

Question 5

An economy has three industries- fishing, forestry and boat building. To produce one tone of fish requires the services of α fishing boats. To produce one ton of timber requires β tones of fish in order to feed the foresters. To produce one ton of fishing boat requires γ tons of timber. Suppose the final demands for the three goods are d_1, d_2 and d_3 85, 95, and 20 units, respectively. If x_1, x_2 and x_3 denote the number of units that have to be produced in the three sectors,

(a) Write down the Leontief model for the problem. **[1]**

(b) Find the number of units that has to be produced in each sector in order to meet the final demands. **[7]**

(c) What assumption would make sense for this economy to achieve efficient level of production of the three commodities? **[2]**

(d) Find the number of units that has to be produced in each sector if the final demands 85, 95, and 20 units, respectively. **[6]**

(d) Using a numerical illustration, briefly define and explain the Leontief model and show how the importance of this concept in Economics **[4]**

Question 6

(i) Consider the simple macroeconomic model described by the three equations:

$Y = C + 10$, $\qquad C = 5 + 0.8(Y - T)$, $\qquad T = 2 + 0.1Y$

Where Y is income, is consumption, T is tax revenue, 10 is the constant (exogenous) autonomous expenditure, and 5,0.8, 2 and 0.1 are all positive parameters. Find the equilibrium values of the endogenous variables Y, C and T by:

(a) successive elimination or substitution [5]

(b) Writing the equations in matrix form and applying Cramer's rule. [5]

(ii) In a standard macroeconomic model for determining national income in a closed economy, it is assumed that

$$Y = C + I \quad \text{(i)} \qquad \text{and} \quad C = f(Y) \quad \text{(ii)}$$

Where (i) states that the national income Y is divided up between consumption (C) and investment (I) whereas (ii) is the consumption function. Assume that the marginal propensity to consume is between o and 1,

(a) Suppose that $C = f(Y) = 95.05 + 0.712\,Y$, use equation (i) and (ii) to find Y in terms of I [5]

(b) Inserting the expression for C from (ii) into (i) gives $Y = f(Y) + I$. Suppose that this equation defines Y as a differentiable function of I, find the expression for dY/dI [5]

End of paper