AFRICA UNIVERSITY
(A United Methodist-Related Institution)

"Investing in Africa's Future"

COLLEGE OF BUSINESS PEACE LEADERSHIP AND GOVERNANCE

QUANTITATIVE ANALYSIS (MMS 202)

FINAL EXAMINATION 2017

MR J. TENGWI

3 HOURS

Instructions:

Answer any five questions

Each question carries a total of 20 marks

No. 1

a. Differentiate between the following terms giving examples in each case:

i. Nominal and ordinal scale [2]

ii. Discrete and continuous data [2]

b. State any three advantages and two disadvantages of using a questionnaire as a method of collecting data [5]

c. Clearly explain the differences between random sampling and non-random sampling [3]

d. Giving an example discuss how you would carry out stratified random sampling and also state any one advantage and disadvantage of this method of sampling. [8]

No.2

The ages of 25 patients admitted to a certain hospital during a particular week were as follows:

67, 74, 69, 54, 26, 37, 67, 68, 73, 54, 52, 72, 55, 75, 37, 24, 13, 12, 43, 26, 39, 24, 54, 47, 44

(a) Find the mean of the data.[2]

(b) Construct an ordered stem and leaf display for the above data. [4]

(c) Using the results in (b) or otherwise to estimate the

i. Median [2]

ii. Lower quartile and [2]

iii. Upper quartile of the data [2]

iv. The mode [2]

(d) Draw a box and whisker plot of the data and comment on the distribution of the data [4]

(e) State any two advantages of using a stem and leaf diagram to represent data.[2]

No. 3

The scores obtained by 40 students in a mathematics test were:

85 81 55 72 86 88 72 86 88 76
72 68 84 84 62 97 70 78 83 94
70 60 91 76 83 94 65 72 92 80
97 76 87 75 95 88 91 82 73 81

(a) Construct a frequency distribution tables using the following intervals:

$50 \leq x < 60,\ 60 \leq x < 70,\ 70 \leq x < 80, 80 \leq x < 90$ *and* $50 \leq x < 100$ [3]

(b) Draw a histogram of the data and use it to estimate the mode of the data. [3]

(c) Use your frequency distribution table to estimate the following:

i. The mean [3]

ii. The standard deviation [5]

iii. The median [4]

iv. Coefficient of variation [2]

No. 4

a. Explain the following terms
   i. Sample space [2]
   ii. Event [2]
   iii. Mutually exclusive events [2]
   iv. Independent events [2]

b. A bag contains three green balls and five yellow balls. One ball is chosen at random and its colour is noted before being replaced in the bag. A second ball is picked and its colour is also noted.
   i. Draw a probability three diagram to show this information. [2]
   ii. Find the probability that two green balls are chosen [2]
   iii. The probability that the two balls are of different colours [2]

c.

i. The probability distribution of a discrete random variable (X) is tabled below.

x	0	1	2	3	4
P(X=x)	0.15	0.15	2p	p	0.1

Find:

a. the value of p [2]
b. $P(X \geq 2)$ [2]
c. E(X) [2]

No. 5

(a) *Consumer Report* states that approximately 70% of all people who buy eyeglasses from a private doctor's office were highly satisfied. In a sample of 10 people buying eyeglasses from a private doctor, what is the probability that :
(i) less than 5 are highly satisfied
(ii) exactly 4 are satisfied
(iii) between 3 and seven are satisfied [6]

(b) The number of radioactive particles emitted in a minute from a meteorite is recorded on a Geiger counter. The mean number is found to be 3.6 per minute. Using the Poisson distribution, find the probability that in any one minute there are
i. no particles
ii. three particles
iii. at least 5 particles [6]

(c) Suppose that the daily demand for change (meaning coins) in a particular store is approximately normally distributed with mean \$800.00 and standard deviation \$60.00. What is the probability that, on any particular day, the demand for change will
(i) be below \$600?
(ii) More than \$900
(iii) Between \$700 and \$900 [8]

No. 6

a. In a study the following sample measurements were obtained.:

110, 362, 246, 85, 510, 208, 173, 425, 316, 179

i. Find a point estimate of the mean of the population[1]

ii. Find a point estimate of the variance of the variance of the population[1]

iii. Find a 90% confidence interval for the population mean [4]

b. The average number of loaves of bread sold by a baker is normally distributed with a standard deviation of 300. The baker claims that on average 3000 loaves are sold daily. An employer wants to test the accuracy of this statement. A random sample of 36 days showed the average daily sales were 3150 loaves. Test at 1% level of significance if the baker's statement is true. [6]

c. Regarding a particular proposal involving the legality of abortion, men and women responded as indicated in the table below.

Gender	Opinion		
	In favour	Opposed	Undecided
Men	86	74	40
Women	119	65	22

Test the hypothesis that there is no difference between mens' and womens' opinions. Use a 5% level of significance. [8]

END OF PAPER

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List of Formulae and Tables for Quantitative Methods MMS 202

1. Measures of central tendency

Ungrouped Data

Sample mean $\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$ where $x_1, x_2, x_3 \ldots\ldots x_n$ are observations and n are the total number of observations.

Population mean $\mu = \frac{1}{N}\sum_{i=1}^{N} x_i$ where $x_1, x_2, x_3 \ldots\ldots x_N$ are measurements and N are the total number of measurements.

Geometric mean = $\sqrt[n]{x_1 x_2 \ldots\ldots x_n}$ where $x_1, x_2, \ldots\ldots, x_n$ are observations.

Harmonic mean = $\frac{n}{\sum_{i=1}^{n}\frac{1}{x_i}}$ where $x_1, x_2, \ldots\ldots, x_n$ are observations.

Grouped Data

Sample mean $\bar{x} = \frac{1}{n}\sum_{i=1}^{n} f_i x_i$

Population mean $\mu = \frac{1}{N}\sum_{i=1}^{N} f_i x_i$

Median $= L_m + \frac{C_m(\frac{n}{2} - F_{m-1})}{f_m}$ where:

L_m = lower limit of median class

C_m = width of median class

f_m = frequency of median class

F_{m-1} = cumulative frequency of the class before the median class.

Mode $= L_m + \frac{C_m(f_m - f_{m-1})}{2f_m - (f_{m-1} + f_{m+1})}$ where:

L_m = lower limit of modal class

C_m = width of modal class

f_m = frequency of modal class

f_{m-1} frequency of the class immediately below modal class

f_{m+1} frequency of the class immediately above modal class

2. Measures of dispersion

Ungrouped Data

Range= Maximum value –Minimum value

Sample Variance $s^2=\frac{1}{n-1}\sum_{i=1}^{n}(x_i-\bar{x})^2$ or $\frac{1}{n-1}\sum_{i=1}^{n}(x_i^2-\frac{(\sum x_i)^2}{n})$

Population Variance $\sigma^2=\frac{1}{N}\sum_{i=1}^{n}(x_i-\mu)^2$ or $\frac{1}{N-1}\sum_{i=1}^{n}(x_i^2-\frac{(\sum x_i)^2}{N})$

Grouped Data with k classes

Sample Variance = $\frac{1}{n-1}\sum_{i=1}^{n}(f_ix_i^2-\frac{(\sum f_ix_i)^2}{n})$

Population Variance = $\frac{1}{N}\sum_{i=1}^{n}(f_ix_i^2-\frac{(\sum f_ix_i)^2}{N})$

Coefficient of variation for the population $=\frac{\sigma}{\mu}$ x100%

Coefficient of variation for the sample $=\frac{s}{\bar{x}}$ x100%

Expectation of a discrete random variable $E(X)=\mu=\sum x_i * P(X=x_i)$

Variance $=\sigma^2 = Var(X) = E(X^2)-\mu^2$

Binomial Distribution function X~B(n;p) $P(X=r) = {}^nC_r\, p^r(1-p)^{n-r}$ where r=0,1,2.....n.

Poisson Distribution function X~P(λ) $P(X=r)=\frac{e^{-\lambda}}{r!}\lambda^r$ where r=0,1,2.....n.

Let X~N(μ,σ^2) then $Z=\frac{X-\mu}{\sigma}$

Confidence limits for the mean:

Large sample $\bar{x}\pm z_{\alpha/2}*\frac{s}{\sqrt{n}}$

Small sample $\bar{x}\pm t_{\alpha/2(n-1)}*\frac{s}{\sqrt{n}}$

Sample size $\left[\frac{z_{\alpha/2}*\sigma}{e}\right]^2$

Testing of Hypothesis

Testing the mean for a single population

i. If σ^2 is unknown, and n is small (i.e. n<30) then the statistic T is such that : $T=\frac{\bar{x}-\mu}{\frac{s}{\sqrt{n}}}$

ii. If σ^2 is known, then the statistic Z is such that: $Z=\frac{\bar{x}-\mu}{\frac{\sigma}{\sqrt{n}}}$

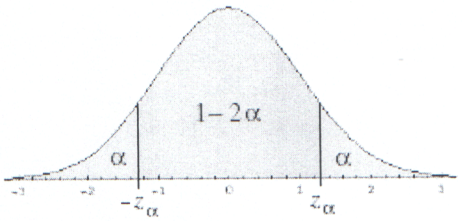
iii. If n is large then the statistic Z is such that: $Z=\frac{\bar{x}-\mu}{\frac{s}{\sqrt{n}}}$

Chi –Square Test

Chi –Square Statistic: $\chi^2=\sum\frac{(O_f-E_f)^2}{E_f}$

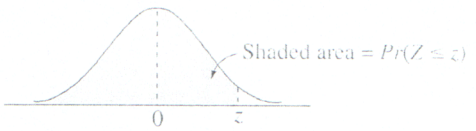
Common confidence levels and their critical values

Confidence Level	Critical Value (Z-score)
0.90	1.645
0.95	1.96
0.99	2.575



Level of significance α	Z_α
0.10	$Z_{.10}$ = **1.28**
0.05	$Z_{.05}$ = **1.645**
0.025	$Z_{.025}$ = **1.96**
0.01	$Z_{.01}$ = **2.33**
0.005	$Z_{.005}$ = **2.58**

TABLE 1
Standard normal curve areas



z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
−3.4	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0002
−3.3	0.0005	0.0005	0.0005	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0003
−3.2	0.0007	0.0007	0.0006	0.0006	0.0006	0.0006	0.0006	0.0005	0.0005	0.0005
−3.1	0.0010	0.0009	0.0009	0.0009	0.0008	0.0008	0.0008	0.0008	0.0007	0.0007
−3.0	0.0013	0.0013	0.0013	0.0012	0.0012	0.0011	0.0011	0.0011	0.0010	0.0010
−2.9	0.0019	0.0018	0.0018	0.0017	0.0016	0.0016	0.0015	0.0015	0.0014	0.0014
−2.8	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022	0.0021	0.0021	0.0020	0.0019
−2.7	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028	0.0027	0.0026
−2.6	0.0047	0.0045	0.0044	0.0043	0.0041	0.0040	0.0039	0.0038	0.0037	0.0036
−2.5	0.0062	0.0060	0.0059	0.0057	0.0055	0.0054	0.0052	0.0051	0.0049	0.0048
−2.4	0.0082	0.0080	0.0078	0.0075	0.0073	0.0071	0.0069	0.0068	0.0066	0.0064
−2.3	0.0107	0.0104	0.0102	0.0099	0.0096	0.0094	0.0091	0.0089	0.0087	0.0084
−2.2	0.0139	0.0136	0.0132	0.0129	0.0125	0.0122	0.0119	0.0116	0.0113	0.0110
−2.1	0.0179	0.0174	0.0170	0.0166	0.0162	0.0158	0.0154	0.0150	0.0146	0.0143
−2.0	0.0228	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197	0.0192	0.0188	0.0183
−1.9	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	0.0244	0.0239	0.0233
−1.8	0.0359	0.0351	0.0344	0.0336	0.0329	0.0322	0.0314	0.0307	0.0301	0.0294
−1.7	0.0446	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392	0.0384	0.0375	0.0367
−1.6	0.0548	0.0537	0.0526	0.0516	0.0505	0.0495	0.0485	0.0475	0.0465	0.0455
−1.5	0.0668	0.0655	0.0643	0.0630	0.0618	0.0606	0.0594	0.0582	0.0571	0.0559
−1.4	0.0808	0.0793	0.0778	0.0764	0.0749	0.0735	0.0721	0.0708	0.0694	0.0681
−1.3	0.0968	0.0951	0.0934	0.0918	0.0901	0.0885	0.0869	0.0853	0.0838	0.0823
−1.2	0.1151	0.1131	0.1112	0.1093	0.1075	0.1056	0.1038	0.1020	0.1003	0.0985
−1.1	0.1357	0.1335	0.1314	0.1292	0.1271	0.1251	0.1230	0.1210	0.1190	0.1170
−1.0	0.1587	0.1562	0.1539	0.1515	0.1492	0.1469	0.1446	0.1423	0.1401	0.1379
−0.9	0.1841	0.1814	0.1788	0.1762	0.1736	0.1711	0.1685	0.1660	0.1635	0.1611
−0.8	0.2119	0.2090	0.2061	0.2033	0.2005	0.1977	0.1949	0.1922	0.1894	0.1867
−0.7	0.2420	0.2389	0.2358	0.2327	0.2296	0.2266	0.2236	0.2206	0.2177	0.2148
−0.6	0.2743	0.2709	0.2676	0.2643	0.2611	0.2578	0.2546	0.2514	0.2483	0.2451
−0.5	0.3085	0.3050	0.3015	0.2981	0.2946	0.2912	0.2877	0.2843	0.2810	0.2776
−0.4	0.3446	0.3409	0.3372	0.3336	0.3300	0.3264	0.3228	0.3192	0.3156	0.3121
−0.3	0.3821	0.3783	0.3745	0.3707	0.3669	0.3632	0.3594	0.3557	0.3520	0.3483
−0.2	0.4207	0.4168	0.4129	0.4090	0.4052	0.4013	0.3974	0.3936	0.3897	0.3859
−0.1	0.4602	0.4562	0.4522	0.4483	0.4443	0.4404	0.4364	0.4325	0.4286	0.4247
−0.0	0.5000	0.4960	0.4920	0.4880	0.4840	0.4801	0.4761	0.4721	0.4681	0.4641

z	Area
−3.50	0.00023263
−4.00	0.00003167
−4.50	0.0000034
−5.00	0.00000029

Source: Computed by M. Longnecker using Splus.

TABLE 1
Standard normal curve areas

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998

z	Area
3.50	0.99976737
4.00	0.99996833
4.50	0.99999660
5.00	0.99999971

t Table

cum. prob	$t_{.50}$	$t_{.75}$	$t_{.80}$	$t_{.85}$	$t_{.90}$	$t_{.95}$	$t_{.975}$	$t_{.99}$	$t_{.995}$	$t_{.999}$	$t_{.9995}$
one-tail	0.50	0.25	0.20	0.15	0.10	0.05	0.025	0.01	0.005	0.001	0.0005
two-tails	1.00	0.50	0.40	0.30	0.20	0.10	0.05	0.02	0.01	0.002	0.001
df											
1	0.000	1.000	1.376	1.963	3.078	6.314	12.71	31.82	63.66	318.31	636.62
2	0.000	0.816	1.061	1.386	1.886	2.920	4.303	6.965	9.925	22.327	31.599
3	0.000	0.765	0.978	1.250	1.638	2.353	3.182	4.541	5.841	10.215	12.924
4	0.000	0.741	0.941	1.190	1.533	2.132	2.776	3.747	4.604	7.173	8.610
5	0.000	0.727	0.920	1.156	1.476	2.015	2.571	3.365	4.032	5.893	6.869
6	0.000	0.718	0.906	1.134	1.440	1.943	2.447	3.143	3.707	5.208	5.959
7	0.000	0.711	0.896	1.119	1.415	1.895	2.365	2.998	3.499	4.785	5.408
8	0.000	0.706	0.889	1.108	1.397	1.860	2.306	2.896	3.355	4.501	5.041
9	0.000	0.703	0.883	1.100	1.383	1.833	2.262	2.821	3.250	4.297	4.781
10	0.000	0.700	0.879	1.093	1.372	1.812	2.228	2.764	3.169	4.144	4.587
11	0.000	0.697	0.876	1.088	1.363	1.796	2.201	2.718	3.106	4.025	4.437
12	0.000	0.695	0.873	1.083	1.356	1.782	2.179	2.681	3.055	3.930	4.318
13	0.000	0.694	0.870	1.079	1.350	1.771	2.160	2.650	3.012	3.852	4.221
14	0.000	0.692	0.868	1.076	1.345	1.761	2.145	2.624	2.977	3.787	4.140
15	0.000	0.691	0.866	1.074	1.341	1.753	2.131	2.602	2.947	3.733	4.073
16	0.000	0.690	0.865	1.071	1.337	1.746	2.120	2.583	2.921	3.686	4.015
17	0.000	0.689	0.863	1.069	1.333	1.740	2.110	2.567	2.898	3.646	3.965
18	0.000	0.688	0.862	1.067	1.330	1.734	2.101	2.552	2.878	3.610	3.922
19	0.000	0.688	0.861	1.066	1.328	1.729	2.093	2.539	2.861	3.579	3.883
20	0.000	0.687	0.860	1.064	1.325	1.725	2.086	2.528	2.845	3.552	3.850
21	0.000	0.686	0.859	1.063	1.323	1.721	2.080	2.518	2.831	3.527	3.819
22	0.000	0.686	0.858	1.061	1.321	1.717	2.074	2.508	2.819	3.505	3.792
23	0.000	0.685	0.858	1.060	1.319	1.714	2.069	2.500	2.807	3.485	3.768
24	0.000	0.685	0.857	1.059	1.318	1.711	2.064	2.492	2.797	3.467	3.745
25	0.000	0.684	0.856	1.058	1.316	1.708	2.060	2.485	2.787	3.450	3.725
26	0.000	0.684	0.856	1.058	1.315	1.706	2.056	2.479	2.779	3.435	3.707
27	0.000	0.684	0.855	1.057	1.314	1.703	2.052	2.473	2.771	3.421	3.690
28	0.000	0.683	0.855	1.056	1.313	1.701	2.048	2.467	2.763	3.408	3.674
29	0.000	0.683	0.854	1.055	1.311	1.699	2.045	2.462	2.756	3.396	3.659
30	0.000	0.683	0.854	1.055	1.310	1.697	2.042	2.457	2.750	3.385	3.646
40	0.000	0.681	0.851	1.050	1.303	1.684	2.021	2.423	2.704	3.307	3.551
60	0.000	0.679	0.848	1.045	1.296	1.671	2.000	2.390	2.660	3.232	3.460
80	0.000	0.678	0.846	1.043	1.292	1.664	1.990	2.374	2.639	3.195	3.416
100	0.000	0.677	0.845	1.042	1.290	1.660	1.984	2.364	2.626	3.174	3.390
1000	0.000	0.675	0.842	1.037	1.282	1.646	1.962	2.330	2.581	3.098	3.300
z	0.000	0.674	0.842	1.036	1.282	1.645	1.960	2.326	2.576	3.090	3.291
	0%	50%	60%	70%	80%	90%	95%	98%	99%	99.8%	99.9%
	Confidence Level										

Table: Chi-Square Probabilities

The areas given across the top are the areas to the right of the critical value. To look up an area on the left subtract it from one, and then look it up (ie: 0.05 on the left is 0.95 on the right)

df	0.995	0.99	0.975	0.95	0.90	0.10	0.05	0.025	0.01	0.005
1	---	---	0.001	0.004	0.016	2.706	3.841	5.024	6.635	7.879
2	0.010	0.020	0.051	0.103	0.211	4.605	5.991	7.378	9.210	10.597
3	0.072	0.115	0.216	0.352	0.584	6.251	7.815	9.348	11.345	12.838
4	0.207	0.297	0.484	0.711	1.064	7.779	9.488	11.143	13.277	14.860
5	0.412	0.554	0.831	1.145	1.610	9.236	11.070	12.833	15.086	16.750
6	0.676	0.872	1.237	1.635	2.204	10.645	12.592	14.449	16.812	18.548
7	0.989	1.239	1.690	2.167	2.833	12.017	14.067	16.013	18.475	20.278
8	1.344	1.646	2.180	2.733	3.490	13.362	15.507	17.535	20.090	21.955
9	1.735	2.088	2.700	3.325	4.168	14.684	16.919	19.023	21.666	23.589
10	2.156	2.558	3.247	3.940	4.865	15.987	18.307	20.483	23.209	25.188
11	2.603	3.053	3.816	4.575	5.578	17.275	19.675	21.920	24.725	26.757
12	3.074	3.571	4.404	5.226	6.304	18.549	21.026	23.337	26.217	28.300
13	3.565	4.107	5.009	5.892	7.042	19.812	22.362	24.736	27.688	29.819
14	4.075	4.660	5.629	6.571	7.790	21.064	23.685	26.119	29.141	31.319
15	4.601	5.229	6.262	7.261	8.547	22.307	24.996	27.488	30.578	32.801
16	5.142	5.812	6.908	7.962	9.312	23.542	26.296	28.845	32.000	34.267
17	5.697	6.408	7.564	8.672	10.085	24.769	27.587	30.191	33.409	35.718
18	6.265	7.015	8.231	9.390	10.865	25.989	28.869	31.526	34.805	37.156
19	6.844	7.633	8.907	10.117	11.651	27.204	30.144	32.852	36.191	38.582
20	7.434	8.260	9.591	10.851	12.443	28.412	31.410	34.170	37.566	39.997
21	8.034	8.897	10.283	11.591	13.240	29.615	32.671	35.479	38.932	41.401
22	8.643	9.542	10.982	12.338	14.041	30.813	33.924	36.781	40.289	42.796
23	9.260	10.196	11.689	13.091	14.848	32.007	35.172	38.076	41.638	44.181
24	9.886	10.856	12.401	13.848	15.659	33.196	36.415	39.364	42.980	45.559
25	10.520	11.524	13.120	14.611	16.473	34.382	37.652	40.646	44.314	46.928
26	11.160	12.198	13.844	15.379	17.292	35.563	38.885	41.923	45.642	48.290
27	11.808	12.879	14.573	16.151	18.114	36.741	40.113	43.195	46.963	49.645
28	12.461	13.565	15.308	16.928	18.939	37.916	41.337	44.461	48.278	50.993
29	13.121	14.256	16.047	17.708	19.768	39.087	42.557	45.722	49.588	52.336
30	13.787	14.953	16.791	18.493	20.599	40.256	43.773	46.979	50.892	53.672
40	20.707	22.164	24.433	26.509	29.051	51.805	55.758	59.342	63.691	66.766
50	27.991	29.707	32.357	34.764	37.689	63.167	67.505	71.420	76.154	79.490
60	35.534	37.485	40.482	43.188	46.459	74.397	79.082	83.298	88.379	91.952