



“Investing in Africa’s Future”

COLLEGE OF BUSINESS, PEACE, LEADERSHIP AND GOVERNANCE

NCSC 103: CALCULUS

END OF FIRST SEMESTER EXAMINATIONS

NOVEMBER 2022

LECTURER: MR A.C MUZENDA

DURATION: 3 HOURS

INSTRUCTIONS

Answer all Questions in Section A and **any three questions from Section B**
Total possible mark is 100

Start **each** question on a new page in your answer Booklet.

The marks allocated to **each** question are shown at the end of the section.

Credit will be awarded for logical, systematic and neat presentation.

Calculators can be used

SECTION A: ANSWER ALL QUESTIONS

QUESTION 1

- a. Discuss and evaluate the methods of evaluating limits. [10 marks]
- b. A cup of coffee is initially 170 degrees Fahrenheit and is left in a room with ambient temperature 70 degrees Fahrenheit. Suppose that when the coffee is first placed in the room, it is cooling at a rate of 20 degrees per minute. Assuming Newton's law of cooling applies, how long does it take for the coffee to cool to 110 degrees? [10 marks]

c. Find the derivatives for;

$$x^4 + x^2y^3 - y^5 = 2x + 1 \quad [5 \text{ marks}]$$

d. Evaluate $\int x \cos x \, dx$ [5 marks]

e. Evaluate the following limits. Show your work

i. $\lim_{x \rightarrow 1} \frac{2 - \sqrt{3+x}}{x-1}$ [5 marks]

ii. $\lim_{x \rightarrow 0} \frac{e^{3x}}{\cos(3x)}$ [5 marks]

SECTION B

Answer any three questions

QUESTION 2

a. Find dy/dx in terms of t for: $x = te^{-2t}$ and $y = t + t^3$ and then find the slope of the curve defined by $x = te^{-2t}$ and $y = t + t^3$ at point $(e^{-2}, 2)$. [10 marks]

b. Evaluate

$$\int x^2 e^{x^2} \, dx \quad [6 \text{ marks}]$$

c. Find $\frac{dy}{dx}$

$$y = \frac{x+5}{x-3} \quad [4 \text{ marks}]$$

QUESTION 3

a) Solve the (separable) differential equations

i. $\frac{dy}{dx} = \frac{x^2 y - 4y}{x + 2}$ [5 marks]

b. Find the area under the curve $y = 5 - 2x^2$ from $x = 0$ to $x = 1$. [5 marks]

- c. Discuss using examples circumstances where you can use the substitution method and integration by parts during integration. [10 marks]

QUESTION 4

a. Prove that $\int_0^{\frac{\pi}{2}} \ln \sin x \, dx = -\frac{\pi}{2} \ln 2$ [8 marks]

- b. Find the limit:

$$\lim_{x \rightarrow 1} \frac{x^2 - 4x + 3}{x^2 - 1} \quad [5 \text{ marks}]$$

- c. Find the absolute maximum and minimum values of $f(x) = 4x^3 - 3x^2$ on the closed interval $(-1, 1)$. [7 marks]

QUESTION 5

a. $\lim_{x \rightarrow 0} \frac{x e^x \cos x - \tan x}{x^2}$ (Use the Hospital's Rule (LHR)) [10 marks]

- b. How many inflection points does the curve $y = 4x^5 - 5x^4 - 12$ have? [6 marks]

- c. Evaluate

$$\lim_{x \rightarrow 2} \frac{\sqrt{x^3 + 8}}{2x + 1} \quad [4 \text{ marks}]$$

THE END