COLLEGE OF BUSINESS, PEACE, LEADERSHIP AND GOVERNANCE

NCSC 201: DISCRETE MATHEMATICS

END OF FIRST SEMESTER EXAMINATIONS

NOVEMBER 2022

LECTURER: MR TIMOTHY MAKAMBWA

TIME: 3 HOURS

INSTRUCTIONS

You are required to answer questions as instructed in each section

Start **each** question on a new page in your answer booklet

Answer all questions in Section **A** and any three from Section **B**

Credit will be awarded for logical, systematic and neat presentations

Section A :(40 marks)

Answer all questions in this Section

Question One

Suppose $a = 5880$ and $b = 8316$.

a. Express a and b as products of primes. [3]

b. Find gcd (a, b) and lcm(a, b). [3]

c. Verify that lcm$(a, b) = |ab| /$ gcd(a, b). [4]

Question Two

Prove that

a. The sum of two even integers is even, [3]

b. The sum of an even integer with and odd integer is odd, [3]

c. The product of two even integers is divisible by 4, [4]

d. The product of an even integer and an odd integer is even. [4]

e. Prove that if n is an integer, then $n^3 - n$ is always divisible by 6 [6]

Question Three

Proof that: $(p \rightarrow r) \vee (q \rightarrow r) \equiv (p \wedge q) \rightarrow r$ .Using the truth table below

p	q	r	$p \rightarrow r$	$q \rightarrow r$	$(p \rightarrow r) \vee (q \rightarrow r)$	$p \wedge q$	$(p \wedge q) \rightarrow r$
0	0	0					
0	0	1					
0	1	0					
0	1	1					
1	0	0					
1	0	1					
1	1	0					
1	1	1					

[10]

Section B (60 marks)

Question Four

Answer any three questions from this Section

i. Show that if n is an integer and $n^3 + 5$ is odd, then n is even using

a. A proof by contraposition. [5]

b. A proof by contradiction. [5]

ii. Prove that if *n* is an integer and $3n + 2$ is even, then *n* is even using

a. A proof by contraposition. [5]

b. A proof by contradiction [5]

Question Five

a. Consider the two integers 125 and 962.

i. Write down the prime decomposition of each of the two numbers.

ii. Find their greatest common divisor.

iii. Find their least common multiple.

b. Factorize the number 6469693230.

c. Find (210; 858). Determine integers x and y such that (210; 858) = 210x + 858y. Hence give the general solution of the equation in integers x and y.

d. Find (182; 247). Determine integers x and y such that (182; 247) = 182x + 247y. Hence give the general solution of the equation in integers x and y.

[20]

Question Six

a. In a survey of university students, 64 had taken mathematics course, 94 had taken chemistry course, 58 had taken physics course, 28 had taken mathematics and physics, 26 had taken mathematics and chemistry, 22 had taken chemistry and physics course, and 14 had taken all the three courses. Find how many had taken one course only. [6]

b. In a group of students, 65 play football, 45 play hockey, 42 play cricket, 20 play football and hockey, 25 play football and cricket, 15 play hockey and cricket and 8 play all the three games. Find the total number of students in the group (Assume that each student in the group plays at least one game.) [6]

c. In a town 85% of the people speak Tamil, 40% speak English and 20% speak Hindi. Also 32% speak Tamil and English, 13% speak Tamil and Hindi and 10% speak English and Hindi, find the percentage of people who can speak all the three languages. [8]

Question Seven

a. Solve the recurrence relation $a_n=7a_{n-1}-10a_{n-2}$ with $a_0=2$ and $a_1=3$. [6]

b. Solve the recurrence relation $a_n=6a_{n-1}-9a_{n-2}$ with initial conditions $a_0=1$ and $a_1=4$.[6]

c. Solve the recurrence relation $a_n=2a_{n-1}-a_{n-2}$

i. What is the solution if the initial terms are $a_0=1$ and $a_1=2$?

ii. What do the initial terms need to be in order for $a_9=30$?

iii. For which xx are there initial terms which make $a_9=x$? [8]

Question Eight

a. Prove that 7^n-1 is a multiple of 6 for all $n \in N$. [5]

b. Prove that $1+3+5+\cdots+(2n-1) = n^2$, for all $n \geq 1$. [5]

c. Prove that $F_0+F_2+F_4+\cdots+F_{2n}=F_{2n+1}-1$, where F_n is the nth Fibonacci number. [5]

d. Prove that $2^n < n!$ For all $n \geq 4$ (Recall, $n! = 1\cdot 2\cdot 3\cdot\cdots\cdot n$.) [5]

END OF PAPER