**“Investing in Africa’s Future”**

**COLLEGE OF BUSINESS, PEACE, LEADERSHIP AND GOVERNANCE**

**QUANTITATIVE ANALYSIS FOR MANAGERS (NMBA503)**

**FINAL EXAMINATION**

**NOVEMBER 2022**

**PROF S. MURAIRWA (PhD)**

**3 HOURS**

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# INSTRUCTIONS

Answer **All** questions in Section A and any **Two (2)** Questions in Section B.

Start **each** question on a new page in your answer booklet.

The marks allocated to **each** question are shown at the end of the question.

**Show all your workings.**

Credit will be given for logical, systematic and neat presentations.

## SECTION A: ANSWER ALL QUESTIONS

1. With examples, discuss the following quantitative decision analysis techniques:
   a) Laplace criterion **[4 marks]**
   b) Savage criterion **[4 marks]**

   c) Hurwicz criterion **[4 marks]**

2. As a first step towards planning new facilities at one of its city centre ticket offices, an airline has collected data on the length of time customers spend at a ticket desk (the service time). One hundred customers were investigated and the time in minutes each one spent at an enquiry desk was measured. The recorded data are shown below.

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 0.9 | 3.5 | 0.8 | 1.0 | 1.3 | 2.3 | 1.0 | 2.4 | 0.7 | 1.0 |
| 2.3 | 0.2 | 1.6 | 1.7 | 5.2 | 1.1 | 3.9 | 5.4 | 8.2 | 1.5 |
| 1.1 | 2.8 | 1.6 | 3.9 | 3.8 | 6.1 | 0.3 | 1.1 | 2.4 | 2.6 |
| 4.0 | 4.3 | 2.7 | 0.2 | 0.3 | 3.1 | 2.7 | 4.1 | 1.4 | 1.1 |
| 3.4 | 0.9 | 2.2 | 4.2 | 2.7 | 3.1 | 1.0 | 3.3 | 3.3 | 5.5 |
| 0.9 | 4.5 | 3.5 | 1.2 | 0.7 | 4.6 | 4.8 | 2.6 | 0.5 | 3.6 |
| 6.3 | 1.6 | 5.0 | 2.1 | 5.8 | 7.4 | 1.7 | 3.8 | 4.1 | 6.9 |
| 3.5 | 2.1 | 0.8 | 7.8 | 1.9 | 3.2 | 1.3 | 1.4 | 3.7 | 0.6 |
| 1.0 | 7.5 | 1.2 | 2.0 | 2.0 | 8.0 | 2.9 | 6.5 | 2.0 | 8.6 |
| 1.5 | 1.2 | 2.9 | 2.9 | 2.0 | 4.6 | 6.6 | 0.7 | 5.8 | 2.0 |

   a) Construct a frequency distribution table for the data **[8 Marks]**
   b) Based on the frequency distribution table you obtained in (a),
      i) Determine the relationship of the measures of central location. Sketch the graph of the relationship. **[9 Marks]**
      ii) Calculate the coefficient of skewness and interpret. **[5 Marks]**
   c) Discuss the quantitative data collection techniques used by the company. **[6 Marks]**

3. A sample of 500 donations to the Arthritis Foundation is reported in the following frequency distribution:

| Amount Spent | Frequency |
|---|---|
| <$6 | 20 |
| $6 up to $8 | 60 |
| $8 up to $10 | 140 |
| $10 up to $12 | 120 |
| $12 up to $14 | 90 |
| >$14 | 70 |

Is it reasonable to conclude that the distribution is normally distributed with a mean of $10 and a standard deviation of $2 at a 5% level of significance? **[10 marks]**

## SECTION B: ANSWER ANY TWO (2) QUESTIONS

4. Briefly discuss the sampling designs. What are the differences between probability and non-probability sampling designs? [**25 marks**]

5. AB Enterprises is a small company that sells freezers. The company employs two salespersons and the manager at AB Enterprises is interested in determining the performance of the two salespersons. Performance data for 50 consecutive days were recorded. The data and analysis outputs are shown below.

| Daily Sales ($) | Salesperson A | Salesperson B |
|---|---|---|
| 60 000 - < 70 000 | 3 | 5 |
| 70 000 - < 80 000 | 13 | 11 |
| 80 000 - < 90 000 | 17 | 22 |
| 90 000 - < 100 000 | 11 | 7 |
| 100 000 - < 110 000 | 4 | 2 |
| 110 000 - < 120 000 | 2 | 3 |

As part of the analysis, the manager calculated the mean, mode and median of the daily performance for A and B salespersons. The summary statistics for A and B salespersons are given below:

| Statistic | Salesperson | |
|---|---|---|
| | A | B |
| Mean | 86 200 | 84000 |
| Median | 85 294 | 85000 |
| Mode | 84 000 | 86 200 |
| Standard Deviation | 12 204 | 11 890 |

Analyse the data. What advise can the manager give to AB Enterprises about the performance of its two salespersons? Justify your response.
[**25 marks**]

6. The output (in kilograms) of BC company was recorded quarterly over four years. The data recorded are presented in the table below.

| | **Quarter** | | | |
|---|---|---|---|---|
| **Year** | Q1 | Q2 | Q3 | Q4 |
| **Y1** | 20 | 10 | 4 | 11 |
| **Y2** | 33 | 17 | 9 | 18 |

| | | | | |
|---|---|---|---|---|
| **Y3** | 45 | 23 | 11 | 25 |
| **Y4** | 60 | 30 | 13 | 29 |

The company manager analysed the data in SPSS and produced the following results:

**Model Summary**

| Model | R | R Square | Adjusted R Square | Std. Error of the Estimate | Change Statistics | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | | | | R Square Change | F Change | df1 | df2 | Sig. F Change |
| 1 | 0.422(a) | 0.178 | 0.120 | 13.746 | 0.178 | 3.041 | 1 | 14 | 0.103 |

a Predictors: (Constant), Time

**ANOVA(b)**

| Model | | Sum of Squares | | df | Mean Square | F | Sig. |
|---|---|---|---|---|---|---|---|
| 1 | Regression | 574.600 | | 1 | 574.600 | 3.041 | 0.103(a) |
| | Residual | 2645.150 | | 14 | 188.939 | | |
| | Total | 3219.750 | | 15 | | | |

a Predictors: (Constant), Time
b Dependent Variable: Series

**Coefficients(a)**

| Model | | Unstandardized Coefficients | | Standardized Coefficients | t | Sig. |
|---|---|---|---|---|---|---|
| | | B | Std. Error | Beta | | |
| 1 | (Constant) | 11.325 | 7.208 | | 1.571 | 0.138 |
| | Time | 1.300 | 0.745 | 0.422 | 1.744 | 0.103 |

a Dependent Variable: Series

(a) Explain the variables and the data collection method used. **[8 marks]**
(b) Write down the regression equation and interpret all the data. [**5 marks**]

(c) Calculate the four seasonal components using a multiplicative model. [**8 marks**]

(d) Forecast the company's quarterly output for the 5th year. [**4 marks**]

**End of paper**

## ADDITIONAL INFORMATION

1. Sturge's Rule:

Number of class, $C=1+3.3\log n$

Class width, $i>\frac{range}{C}$

2. Mean of grouped data $=\frac{\sum_{i=1}^{n} fx_i}{n}$

3. Mean of ungrouped data $=\frac{\sum_{i=1}^{n} x_i}{n}$

. Mode = $L_{mo}+\left(\frac{\Delta_1}{\Delta_1+\Delta_2}\right) i$

4

5 Median = $L_{me}+\left(\frac{\frac{n}{2}-F}{f_m}\right) i$

6. Standard deviation: $S=\sqrt{\frac{\sum_{i=1}^{n} fx_i^2-\frac{\left(\sum_{n=1}^{n} fx_i\right)^2}{n}}{n-1}}$

7. Standard Deviation of ungrouped data: $S=\sqrt{\frac{\sum_{i=1}^{n} x_i^2-\frac{\left(\sum_{i=1}^{n} x_i\right)^2}{n}}{n-1}}$

8. Coefficient of skewness: $S_k=\frac{3(mean-median)}{s}=\frac{mean-\mathrm{mod}\,e}{s}$

9. Conditional probability: $P(A ¿)=\frac{P(A\cap B)}{P(A)}$

10. Binomial Distribution

- $P(X=x)=nC_x p^x q^{n-x}$

11. Poisson Distribution

- $P(X=x)=\frac{e^{-\lambda}\lambda^x}{x!}$

12. Hypothesis testing (single mean)

- $Z=\frac{\overline{X}-\mu}{\frac{\sigma}{\sqrt{n}}}$
- $t=\frac{\overline{X}-\mu}{\frac{s}{\sqrt{n}}}$, $df$ = n – 1

13. Hypothesis testing (single proportion)

- $Z=\frac{p-\pi}{\sqrt{\frac{\pi(1-\pi)}{n}}}$

14. Hypothesis testing (difference of two means)

- $Z=\frac{\overline{X}_1-\overline{X}_2}{\sqrt{\frac{\sigma_1^2}{n_1}+\frac{\sigma_2^2}{n_2}}}$
- $t=\frac{\overline{X}_1-\overline{X}_2}{\sqrt{\frac{S_1^2}{n_1}+\frac{S_2^2}{n_2}}}$, $df$ = smaller $(n_1-1; n_2-1)$
- $t=\frac{\overline{X}_1-\overline{X}_2}{\sqrt{\frac{S_p^2}{n_1}+\frac{S_p^2}{n_2}}}$

  Where $S_p^2=\frac{S_1^2(n_1)+S_2^2(n_2)}{n_1+n_2-2}$, $df=n_1+n_2-2$
- $t=\frac{\overline{D}-\mu_D}{\frac{S_D}{\sqrt{n}}}$, $df$ = n – 1

15. Hypothesis testing (difference of two proportions)

- $Z=\frac{p_1-p_2}{\sqrt{\overline{p}\,\overline{q}\left(\frac{1}{n_1}+\frac{1}{n_2}\right)}}$

16. Confidence Interval (Single mean)

- $\overline{X}-Z_{\frac{\alpha}{2}}\frac{\sigma}{\sqrt{n}}\leq\mu\leq\overline{X}+Z_{\frac{\alpha}{2}}\frac{\sigma}{\sqrt{n}}$
- $\overline{X}-t_{\frac{\alpha}{2}}\frac{S}{\sqrt{n}}\leq\mu\leq\overline{X}+t_{\frac{\alpha}{2}}\frac{S}{\sqrt{n}}$

17. Confidence Interval (Difference of two means)

- $(\overline{X}_1-\overline{X}_2)-Z_{\frac{\alpha}{2}}\sqrt{\frac{\sigma_1^2}{n_1}+\frac{\sigma_2^2}{n_2}}\leq(\mu_1-\mu_2)\leq(\overline{X}_1-\overline{X}_2)+Z_{\frac{\alpha}{2}}\sqrt{\frac{\sigma_1^2}{n_1}+\frac{\sigma_2^2}{n_2}}$

- $\left(\overline{X}_1-\overline{X}_2\right)-t_{\frac{\alpha}{2}}\sqrt{\frac{S_1^2}{n_1}+\frac{S_2^2}{n_2}}\leq\left(\mu_1-\mu_2\right)\leq\left(\overline{X}_1-\overline{X}_2\right)+t_{\frac{\alpha}{2}}\sqrt{\frac{S_1^2}{n_1}+\frac{S_2^2}{n_2}}$
  *df* = smaller $\left(n_1-1;n_2-1\right)$
- $\left(\overline{X}_1-\overline{X}_2\right)-t_{\frac{\alpha}{2}}\sqrt{\frac{S_p^2}{n_1}+\frac{S_p^2}{n_2}}\leq\left(\mu_1-\mu_2\right)\leq\left(\overline{X}_1-\overline{X}_2\right)+t_{\frac{\alpha}{2}}\sqrt{\frac{S_p^2}{n_1}+\frac{S_p^2}{n_2}}$
  Where $S_p^2=\frac{S_1^2\left(n_1\right)+S_2^2\left(n_2\right)}{n_1+n_2-2}$, $df=n_1+n_2-2$

18. Confidence Interval (Single proportion)
    - $p-Z_{\frac{\alpha}{2}}\sqrt{\frac{pq}{n}}\leq\pi\leq p+Z_{\frac{\alpha}{2}}\sqrt{\frac{pq}{n}}$

19. Confidence Interval (Difference of two proportions)
    - $\left(p_1-p_2\right)-Z_{\frac{\alpha}{2}}\sqrt{\frac{p_1q_1}{n_1}+\frac{p_2q_2}{n_2}}\leq\left(\pi_1-\mu_2\right)\leq\left(p_1-p_2\right)+Z_{\frac{\alpha}{2}}\sqrt{\frac{p_1q_1}{n_1}+\frac{p_2q_2}{n_2}}$

20. Standardisation: $Z=\frac{X-\mu}{\sigma}$

21. Weighted Mean: $\overline{X}_w=\frac{\sum xw}{\sum w}$

    Bayes Theory: $P\left(B_i/C\right)=\frac{P\left(C/B_i\right)P\left(B_i\right)}{\sum_{i=1}^{n}P\left(C/B_i\right)P\left(B_i\right)}$,

22.

23. Regression and Correlation Analysis:

    $y=\beta_0+\beta_1x+e$,

    $\beta_1=\frac{n\sum xy-\sum x\sum y}{n\sum y^2-\left(\sum y\right)^2}$

    $\beta_0=\overline{y}+\beta_1\overline{x}$

    $r=\frac{n\sum xy-\sum x\sum y}{\sqrt{\left\{\left(n\sum x^2-\left(\sum x\right)^2\right)\left(n\sum y^2-\left(\sum y\right)^2\right)\right\}}}$

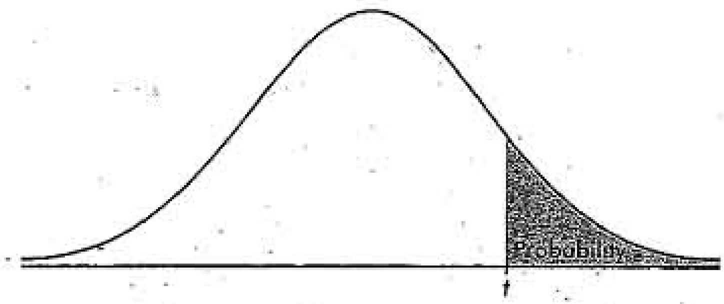
24. Chi-square Test

Let $f_0$ and $f_e$ be the observed and expected frequencies respectively:

$$\chi^2 = \sum \frac{(f_o - f_e)^2}{f_e}$$

# Table of Random Numbers

| | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 36518 | 36777 | 89116 | 05542 | 29705 | 83775 | 21564 | 81639 | 27973 | 62413 | 85652 | 62817 | 57881 |
| 46132 | 81380 | 75635 | 19428 | 88048 | 08747 | 20092 | 12615 | 35046 | 67753 | 69630 | 10883 | 13683 |
| 31841 | 77367 | 40791 | 97402 | 27569 | 90184 | 02338 | 39318 | 54936 | 34641 | 95525 | 86316 | 87384 |
| 84180 | 93793 | 64953 | 51472 | 65358 | 23701 | 75230 | 47200 | 78176 | 85248 | 90589 | 74567 | 22633 |
| 78435 | 37586 | 07015 | 98729 | 76703 | 16224 | 97661 | 79907 | 06611 | 26501 | 93389 | 92725 | 68158 |
| 41859 | 94198 | 37182 | 61345 | 88857 | 53204 | 86721 | 59613 | 67494 | 17292 | 94457 | 89520 | 77771 |
| 13019 | 07274 | 51068 | 93129 | 40386 | 51731 | 44254 | 66685 | 72835 | 01270 | 42523 | 45323 | 63481 |
| 82448 | 72430 | 29041 | 59208 | 95266 | 33978 | 70958 | 60017 | 39723 | 00606 | 17956 | 19024 | 15819 |
| 25432 | 96593 | 83112 | 96997 | 55340 | 80312 | 78839 | 09815 | 16887 | 22228 | 06206 | 54272 | 83516 |
| 69226 | 38655 | 03811 | 08342 | 47863 | 02743 | 11547 | 38250 | 58140 | 98470 | 24364 | 99797 | 73498 |
| 25837 | 68821 | 66426 | 20496 | 84843 | 18360 | 91252 | 99134 | 48931 | 99538 | 21160 | 09411 | 44659 |
| 38914 | 82707 | 24769 | 72026 | 56813 | 49336 | 71767 | 04474 | 32909 | 74162 | 50404 | 68562 | 14088 |
| 04070 | 60681 | 64290 | 26905 | 65617 | 76039 | 91657 | 71362 | 32246 | 49595 | 50663 | 47459 | 57072 |
| 01674 | 14751 | 28637 | 86980 | 11951 | 10479 | 41454 | 48527 | 53868 | 37846 | 85912 | 15156 | 00865 |
| 70294 | 35450 | 39982 | 79503 | 34382 | 43186 | 69890 | 63222 | 30110 | 56004 | 04879 | 05138 | 57476 |
| 73903 | 98066 | 52136 | 89925 | 50000 | 96334 | 30773 | 80571 | 31178 | 52799 | 41050 | 76298 | 43995 |
| 87789 | 56408 | 77107 | 88452 | 80975 | 03406 | 36114 | 64549 | 79244 | 82044 | 00202 | 45727 | 35709 |
| 92320 | 95929 | 58545 | 70699 | 07679 | 23296 | 03002 | 63885 | 54677 | 55745 | 52540 | 62154 | 33314 |
| 46391 | 60276 | 92061 | 43591 | 42118 | 73094 | 53608 | 58949 | 42927 | 90993 | 46795 | 05947 | 01934 |
| 67090 | 45063 | 84584 | 66022 | 48268 | 74971 | 94861 | 61749 | 61085 | 81758 | 89640 | 39437 | 90044 |
| 11666 | 99916 | 35165 | 29420 | 73213 | 15275 | 62532 | 47319 | 39842 | 62273 | 94980 | 23415 | 64668 |
| 40910 | 59068 | 04594 | 94576 | 51187 | 54796 | 17411 | 56123 | 66545 | 82163 | 61868 | 22752 | 40101 |
| 41169 | 37965 | 47578 | 92180 | 05257 | 19143 | 77486 | 02457 | 00985 | 31960 | 39033 | 44374 | 28352 |




## TABLE B: t-DISTRIBUTION CRITICAL VALUES

| | Tail probability $p$ | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| df | .25 | .20 | .15 | .10 | .05 | .025 | .02 | .01 | .005 | .0025 | .001 | .0005 |
| 1 | 1.000 | 1.376 | 1.963 | 3.078 | 6.314 | 12.71 | 15.89 | 31.82 | 63.66 | 127.3 | 318.3 | 636.6 |
| 2 | .816 | 1.061 | 1.386 | 1.886 | 2.920 | 4.303 | 4.849 | 6.965 | 9.925 | 14.09 | 22.33 | 31.60 |
| 3 | .765 | .978 | 1.250 | 1.638 | 2.353 | 3.182 | 3.482 | 4.541 | 5.841 | 7.453 | 10.21 | 12.92 |
| 4 | .741 | .941 | 1.190 | 1.533 | 2.132 | 2.776 | 2.999 | 3.747 | 4.604 | 5.598 | 7.173 | 8.610 |
| 5 | .727 | .920 | 1.156 | 1.476 | 2.015 | 2.571 | 2.757 | 3.365 | 4.032 | 4.773 | 5.893 | 6.869 |
| 6 | .718 | .906 | 1.134 | 1.440 | 1.943 | 2.447 | 2.612 | 3.143 | 3.707 | 4.317 | 5.208 | 5.959 |
| 7 | .711 | .896 | 1.119 | 1.415 | 1.895 | 2.365 | 2.517 | 2.998 | 3.499 | 4.029 | 4.785 | 5.408 |
| 8 | .706 | .889 | 1.108 | 1.397 | 1.860 | 2.306 | 2.449 | 2.896 | 3.355 | 3.833 | 4.501 | 5.041 |
| 9 | .703 | .883 | 1.100 | 1.383 | 1.833 | 2.262 | 2.398 | 2.821 | 3.250 | 3.690 | 4.297 | 4.781 |
| 10 | .700 | .879 | 1.093 | 1.372 | 1.812 | 2.228 | 2.359 | 2.764 | 3.169 | 3.581 | 4.144 | 4.587 |
| 11 | .697 | .876 | 1.088 | 1.363 | 1.796 | 2.201 | 2.328 | 2.718 | 3.106 | 3.497 | 4.025 | 4.437 |
| 12 | .695 | .873 | 1.083 | 1.356 | 1.782 | 2.179 | 2.303 | 2.681 | 3.055 | 3.428 | 3.930 | 4.318 |
| 13 | .694 | .870 | 1.079 | 1.350 | 1.771 | 2.160 | 2.282 | 2.650 | 3.012 | 3.372 | 3.852 | 4.221 |
| 14 | .692 | .868 | 1.076 | 1.345 | 1.761 | 2.145 | 2.264 | 2.624 | 2.977 | 3.326 | 3.787 | 4.140 |
| 15 | .691 | .866 | 1.074 | 1.341 | 1.753 | 2.131 | 2.249 | 2.602 | 2.947 | 3.286 | 3.733 | 4.073 |
| 16 | .690 | .865 | 1.071 | 1.337 | 1.746 | 2.120 | 2.235 | 2.583 | 2.921 | 3.252 | 3.686 | 4.015 |
| 17 | .689 | .863 | 1.069 | 1.333 | 1.740 | 2.110 | 2.224 | 2.567 | 2.898 | 3.222 | 3.646 | 3.965 |
| 18 | .688 | .862 | 1.067 | 1.330 | 1.734 | 2.101 | 2.214 | 2.552 | 2.878 | 3.197 | 3.611 | 3.922 |
| 19 | .688 | .861 | 1.066 | 1.328 | 1.729 | 2.093 | 2.205 | 2.539 | 2.861 | 3.174 | 3.579 | 3.883 |
| 20 | .687 | .860 | 1.064 | 1.325 | 1.725 | 2.086 | 2.197 | 2.528 | 2.845 | 3.153 | 3.552 | 3.850 |
| 21 | .686 | .859 | 1.063 | 1.323 | 1.721 | 2.080 | 2.189 | 2.518 | 2.831 | 3.135 | 3.527 | 3.819 |
| 22 | .686 | .858 | 1.061 | 1.321 | 1.717 | 2.074 | 2.183 | 2.508 | 2.819 | 3.119 | 3.505 | 3.792 |
| 23 | .685 | .858 | 1.060 | 1.319 | 1.714 | 2.069 | 2.177 | 2.500 | 2.807 | 3.104 | 3.485 | 3.768 |
| 24 | .685 | .857 | 1.059 | 1.318 | 1.711 | 2.064 | 2.172 | 2.492 | 2.797 | 3.091 | 3.467 | 3.745 |
| 25 | .684 | .856 | 1.058 | 1.316 | 1.708 | 2.060 | 2.167 | 2.485 | 2.787 | 3.078 | 3.450 | 3.725 |
| 26 | .684 | .856 | 1.058 | 1.315 | 1.706 | 2.056 | 2.162 | 2.479 | 2.779 | 3.067 | 3.435 | 3.707 |
| 27 | .684 | .855 | 1.057 | 1.314 | 1.703 | 2.052 | 2.158 | 2.473 | 2.771 | 3.057 | 3.421 | 3.690 |
| 28 | .683 | .855 | 1.056 | 1.313 | 1.701 | 2.048 | 2.154 | 2.467 | 2.763 | 3.047 | 3.408 | 3.674 |
| 29 | .683 | .854 | 1.055 | 1.311 | 1.699 | 2.045 | 2.150 | 2.462 | 2.756 | 3.038 | 3.396 | 3.659 |
| 30 | .683 | .854 | 1.055 | 1.310 | 1.697 | 2.042 | 2.147 | 2.457 | 2.750 | 3.030 | 3.385 | 3.646 |
| 40 | .681 | .851 | 1.050 | 1.303 | 1.684 | 2.021 | 2.123 | 2.423 | 2.704 | 2.971 | 3.307 | 3.551 |
| 50 | .679 | .849 | 1.047 | 1.299 | 1.676 | 2.009 | 2.109 | 2.403 | 2.678 | 2.937 | 3.261 | 3.496 |
| 60 | .679 | .848 | 1.045 | 1.296 | 1.671 | 2.000 | 2.099 | 2.390 | 2.660 | 2.915 | 3.232 | 3.460 |
| 80 | .678 | .846 | 1.043 | 1.292 | 1.664 | 1.990 | 2.088 | 2.374 | 2.639 | 2.887 | 3.195 | 3.416 |
| 100 | .677 | .845 | 1.042 | 1.290 | 1.660 | 1.984 | 2.081 | 2.364 | 2.626 | 2.871 | 3.174 | 3.390 |
| 1000 | .675 | .842 | 1.037 | 1.282 | 1.646 | 1.962 | 2.056 | 2.330 | 2.581 | 2.813 | 3.098 | 3.300 |
| ∞ | .674 | .841 | 1.036 | 1.282 | 1.645 | 1.960 | 2.054 | 2.326 | 2.576 | 2.807 | 3.091 | 3.291 |
| | 50% | 60% | 70% | 80% | 90% | 95% | 96% | 98% | 99% | 99.5% | 99.8% | 99.9% |
| | Confidence level $C$ | | | | | | | | | | | |

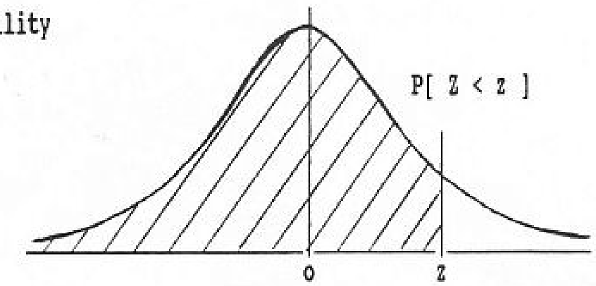
## STANDARD STATISTICAL TABLES

## 1. Areas under the Normal Distribution

The table gives the cumulative probability up to the standardised normal value z

i.e.

$$P[\,Z < z\,] = \int_{-\infty}^{z} \frac{1}{\sqrt{2\pi}} \exp(-\tfrac{1}{2}Z^2)\, dZ$$




| z | 0.00 | 0.01 | 0.02 | 0.03 | 0.04 | 0.05 | 0.06 | 0.07 | 0.08 | 0.09 |
|---|---|---|---|---|---|---|---|---|---|---|
| 0.0 | 0.5000 | 0.5040 | 0.5080 | 0.5120 | 0.5159 | 0.5199 | 0.5239 | 0.5279 | 0.5319 | 0.5359 |
| 0.1 | 0.5398 | 0.5438 | 0.5478 | 0.5517 | 0.5557 | 0.5596 | 0.5636 | 0.5675 | 0.5714 | 0.5753 |
| 0.2 | 0.5793 | 0.5832 | 0.5871 | 0.5910 | 0.5948 | 0.5987 | 0.6026 | 0.6064 | 0.6103 | 0.6141 |
| 0.3 | 0.6179 | 0.6217 | 0.6255 | 0.6293 | 0.6331 | 0.6368 | 0.6406 | 0.6443 | 0.6480 | 0.6517 |
| 0.4 | 0.6554 | 0.6591 | 0.6628 | 0.6664 | 0.6700 | 0.6736 | 0.6772 | 0.6808 | 0.6844 | 0.6879 |
| 0.5 | 0.6915 | 0.6950 | 0.6985 | 0.7019 | 0.7054 | 0.7088 | 0.7123 | 0.7157 | 0.7190 | 0.7224 |
| 0.6 | 0.7257 | 0.7291 | 0.7324 | 0.7357 | 0.7389 | 0.7422 | 0.7454 | 0.7486 | 0.7517 | 0.7549 |
| 0.7 | 0.7580 | 0.7611 | 0.7642 | 0.7673 | 0.7704 | 0.7734 | 0.7764 | 0.7794 | 0.7823 | 0.7854 |
| 0.8 | 0.7881 | 0.7910 | 0.7939 | 0.7967 | 0.7995 | 0.8023 | 0.8051 | 0.8078 | 0.8106 | 0.8133 |
| 0.9 | 0.8159 | 0.8186 | 0.8212 | 0.8238 | 0.8264 | 0.8289 | 0.8315 | 0.8340 | 0.8365 | 0.8389 |
| 1.0 | 0.8413 | 0.8438 | 0.8461 | 0.8485 | 0.8508 | 0.8531 | 0.8554 | 0.8577 | 0.8599 | 0.8621 |
| 1.1 | 0.8643 | 0.8665 | 0.8686 | 0.8708 | 0.8729 | 0.8749 | 0.8770 | 0.8790 | 0.8804 | 0.8830 |
| 1.2 | 0.8849 | 0.8869 | 0.8888 | 0.8907 | 0.8925 | 0.8944 | 0.8962 | 0.8980 | 0.8997 | 0.9015 |
| 1.3 | 0.9032 | 0.9049 | 0.9066 | 0.9082 | 0.9099 | 0.9115 | 0.9131 | 0.9147 | 0.9162 | 0.9177 |
| 1.4 | 0.9192 | 0.9207 | 0.9222 | 0.9236 | 0.9251 | 0.9265 | 0.9279 | 0.9292 | 0.9306 | 0.9319 |
| 1.5 | 0.9332 | 0.9345 | 0.9357 | 0.9370 | 0.9382 | 0.9394 | 0.9406 | 0.9418 | 0.9429 | 0.9441 |
| 1.6 | 0.9452 | 0.9463 | 0.9474 | 0.9484 | 0.9495 | 0.9505 | 0.9515 | 0.9525 | 0.9535 | 0.9545 |
| 1.7 | 0.9554 | 0.9564 | 0.9573 | 0.9582 | 0.9591 | 0.9599 | 0.9608 | 0.9616 | 0.9625 | 0.9633 |
| 1.8 | 0.9641 | 0.9649 | 0.9656 | 0.9664 | 0.9671 | 0.9678 | 0.9686 | 0.9693 | 0.9699 | 0.9706 |
| 1.9 | 0.9713 | 0.9719 | 0.9726 | 0.9732 | 0.9738 | 0.9744 | 0.9750 | 0.9756 | 0.9761 | 0.9767 |
| 2.0 | 0.9773 | 0.9778 | 0.9783 | 0.9788 | 0.9793 | 0.9798 | 0.9803 | 0.9808 | 0.9812 | 0.9817 |
| 2.1 | 0.9821 | 0.9826 | 0.9830 | 0.9834 | 0.9838 | 0.9842 | 0.9846 | 0.9850 | 0.9854 | 0.9857 |
| 2.2 | 0.9861 | 0.9865 | 0.9868 | 0.9871 | 0.9874 | 0.9878 | 0.9881 | 0.9884 | 0.9887 | 0.9890 |
| 2.3 | 0.9893 | 0.9896 | 0.9898 | 0.9901 | 0.9904 | 0.9906 | 0.9909 | 0.9911 | 0.9913 | 0.9916 |
| 2.4 | 0.9918 | 0.9920 | 0.9922 | 0.9924 | 0.9927 | 0.9929 | 0.9931 | 0.9932 | 0.9934 | 0.9936 |
| 2.5 | 0.9938 | 0.9940 | 0.9941 | 0.9943 | 0.9945 | 0.9946 | 0.9948 | 0.9949 | 0.9951 | 0.9952 |
| 2.6 | 0.9953 | 0.9955 | 0.9956 | 0.9957 | 0.9959 | 0.9960 | 0.9961 | 0.9962 | 0.9963 | 0.9964 |
| 2.7 | 0.9965 | 0.9966 | 0.9967 | 0.9968 | 0.9969 | 0.9970 | 0.9971 | 0.9972 | 0.9973 | 0.9974 |
| 2.8 | 0.9974 | 0.9975 | 0.9976 | 0.9977 | 0.9977 | 0.9978 | 0.9979 | 0.9980 | 0.9980 | 0.9981 |
| 2.9 | 0.9981 | 0.9982 | 0.9982 | 0.9983 | 0.9984 | 0.9984 | 0.9985 | 0.9985 | 0.9986 | 0.9986 |

| z | 3.00 | 3.10 | 3.20 | 3.30 | 3.40 | 3.50 | 3.60 | 3.70 | 3.80 | 3.90 |
|---|---|---|---|---|---|---|---|---|---|---|
| P | 0.9986 | 0.9990 | 0.9993 | 0.9995 | 0.9997 | 0.9998 | 0.9998 | 0.9999 | 0.9999 | 1.0000 |

# Chi-square Distribution

Chi-Square ($\chi^2$) Distribution

| Degrees of Freedom | Area to the Right of Critical Value | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | 0.995 | 0.99 | 0.975 | 0.95 | 0.90 | 0.10 | 0.05 | 0.025 | 0.01 | 0.005 |
| **1** | — | — | 0.001 | 0.004 | 0.016 | 2.706 | 3.841 | 5.024 | 6.635 | 7.879 |
| **2** | 0.010 | 0.020 | 0.051 | 0.103 | 0.211 | 4.605 | 5.991 | 7.378 | 9.210 | 10.597 |
| **3** | 0.072 | 0.115 | 0.216 | 0.352 | 0.584 | 6.251 | 7.815 | 9.348 | 11.345 | 12.838 |
| **4** | 0.207 | 0.297 | 0.484 | 0.711 | 1.064 | 7.779 | 9.488 | 11.143 | 13.277 | 14.860 |
| **5** | 0.412 | 0.554 | 0.831 | 1.145 | 1.610 | 9.236 | 11.071 | 12.833 | 15.086 | 16.750 |
| **6** | 0.676 | 0.872 | 1.237 | 1.635 | 2.204 | 10.645 | 12.592 | 14.449 | 16.812 | 18.548 |
| **7** | 0.989 | 1.239 | 1.690 | 2.167 | 2.833 | 12.017 | 14.067 | 16.013 | 18.475 | 20.278 |
| **8** | 1.344 | 1.646 | 2.180 | 2.733 | 3.490 | 13.362 | 15.507 | 17.535 | 20.090 | 21.955 |
| **9** | 1.735 | 2.088 | 2.700 | 3.325 | 4.168 | 14.684 | 16.919 | 19.023 | 21.666 | 23.589 |
| **10** | 2.156 | 2.558 | 3.247 | 3.940 | 4.865 | 15.987 | 18.307 | 20.483 | 23.209 | 25.188 |
| **11** | 2.603 | 3.053 | 3.816 | 4.575 | 5.578 | 17.275 | 19.675 | 21.920 | 24.725 | 26.757 |
| **12** | 3.074 | 3.571 | 4.404 | 5.226 | 6.304 | 18.549 | 21.026 | 23.337 | 26.217 | 28.299 |
| **13** | 3.565 | 4.107 | 5.009 | 5.892 | 7.042 | 19.812 | 22.362 | 24.736 | 27.688 | 29.819 |
| **14** | 4.075 | 4.660 | 5.629 | 6.571 | 7.790 | 21.064 | 23.685 | 26.119 | 29.141 | 31.319 |
| **15** | 4.601 | 5.229 | 6.262 | 7.261 | 8.547 | 22.307 | 24.996 | 27.488 | 30.578 | 32.801 |
| **16** | 5.142 | 5.812 | 6.908 | 7.962 | 9.312 | 23.542 | 26.296 | 28.845 | 32.000 | 34.267 |
| **17** | 5.697 | 6.408 | 7.564 | 8.672 | 10.085 | 24.769 | 27.587 | 30.191 | 33.409 | 35.718 |
| **18** | 6.265 | 7.015 | 8.231 | 9.390 | 10.865 | 25.989 | 28.869 | 31.526 | 34.805 | 37.156 |
| **19** | 6.844 | 7.633 | 8.907 | 10.117 | 11.651 | 27.204 | 30.144 | 32.852 | 36.191 | 38.582 |
| **20** | 7.434 | 8.260 | 9.591 | 10.851 | 12.443 | 28.412 | 31.410 | 34.170 | 37.566 | 39.997 |
| **21** | 8.034 | 8.897 | 10.283 | 11.591 | 13.240 | 29.615 | 32.671 | 35.479 | 38.932 | 41.401 |
| **22** | 8.643 | 9.542 | 10.982 | 12.338 | 14.042 | 30.813 | 33.924 | 36.781 | 40.289 | 42.796 |
| **23** | 9.260 | 10.196 | 11.689 | 13.091 | 14.848 | 32.007 | 35.172 | 38.076 | 41.638 | 44.181 |
| **24** | 9.886 | 10.856 | 12.401 | 13.848 | 15.659 | 33.196 | 36.415 | 39.364 | 42.980 | 45.559 |
| **25** | 10.520 | 11.524 | 13.120 | 14.611 | 16.473 | 34.382 | 37.652 | 40.646 | 44.314 | 46.928 |
| **26** | 11.160 | 12.198 | 13.844 | 15.379 | 17.292 | 35.563 | 38.885 | 41.923 | 45.642 | 48.290 |
| **27** | 11.808 | 12.879 | 14.573 | 16.151 | 18.114 | 36.741 | 40.113 | 43.194 | 46.963 | 49.645 |
| **28** | 12.461 | 13.565 | 15.308 | 16.928 | 18.939 | 37.916 | 41.337 | 44.461 | 48.278 | 50.993 |
| **29** | 13.121 | 14.257 | 16.047 | 17.708 | 19.768 | 39.087 | 42.557 | 45.722 | 49.588 | 52.336 |
| **30** | 13.787 | 14.954 | 16.791 | 18.493 | 20.599 | 40.256 | 43.773 | 46.979 | 50.892 | 53.672 |
| **40** | 20.707 | 22.164 | 24.433 | 26.509 | 29.051 | 51.805 | 55.758 | 59.342 | 63.691 | 66.766 |
| **50** | 27.991 | 29.707 | 32.357 | 34.764 | 37.689 | 63.167 | 67.505 | 71.420 | 76.154 | 79.490 |
| **60** | 35.534 | 37.485 | 40.482 | 43.188 | 46.459 | 74.397 | 79.082 | 83.298 | 88.379 | 91.952 |
| **70** | 43.275 | 45.442 | 48.758 | 51.739 | 55.329 | 85.527 | 90.531 | 95.023 | 100.425 | 104.215 |
| **80** | 51.172 | 53.540 | 57.153 | 60.391 | 64.278 | 96.578 | 101.879 | 106.629 | 112.329 | 116.321 |
| **90** | 59.196 | 61.754 | 65.647 | 69.126 | 73.291 | 107.565 | 113.145 | 118.136 | 124.116 | 128.299 |
| **100** | 67.328 | 70.065 | 74.222 | 77.929 | 82.358 | 118.498 | 124.342 | 129.561 | 135.807 | 140.169 |

## F-Distribution Table

Table A.5. F-distribution where α = 0.05

| $V_1$ \ $V_2$ | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 12 | 15 | 20 | 24 | 30 | 40 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **1** | 161.4 | 199.5 | 215.7 | 224.6 | 230.2 | 234.0 | 236.8 | 238.9 | 240.5 | 241.9 | 243.9 | 245.9 | 248.0 | 249.1 | 250.1 | 251.1 |
| **2** | 18.51 | 19.00 | 19.16 | 19.25 | 19.30 | 19.33 | 19.35 | 19.37 | 19.38 | 19.40 | 19.41 | 19.43 | 19.45 | 19.45 | 19.46 | 19.47 |
| **3** | 10.13 | 9.55 | 9.28 | 9.12 | 9.01 | 8.94 | 8.89 | 8.85 | 8.81 | 8.79 | 8.74 | 8.70 | 8.66 | 8.64 | 8.62 | 8.59 |
| **4** | 7.71 | 6.94 | 6.59 | 6.39 | 6.26 | 6.16 | 6.09 | 6.04 | 6.00 | 5.96 | 5.91 | 5.86 | 5.80 | 5.77 | 5.75 | 5.72 |
| **5** | 6.61 | 5.79 | 5.41 | 5.19 | 5.05 | 4.95 | 4.88 | 4.82 | 4.77 | 4.74 | 4.68 | 4.62 | 4.56 | 4.53 | 4.50 | 4.46 |
| **6** | 5.99 | 5.14 | 4.76 | 4.53 | 4.39 | 4.28 | 4.21 | 4.15 | 4.10 | 4.06 | 4.00 | 3.94 | 3.87 | 3.84 | 3.81 | 3.77 |
| **7** | 5.59 | 4.74 | 4.35 | 4.12 | 3.97 | 3.87 | 3.79 | 3.73 | 3.68 | 3.64 | 3.57 | 3.51 | 3.44 | 3.41 | 3.38 | 3.34 |
| **8** | 5.32 | 4.46 | 4.07 | 3.84 | 3.69 | 3.58 | 3.50 | 3.44 | 3.39 | 3.35 | 3.28 | 3.22 | 3.15 | 3.12 | 3.08 | 3.04 |
| **9** | 5.12 | 4.26 | 3.86 | 3.63 | 3.48 | 3.37 | 3.29 | 3.23 | 3.18 | 3.14 | 3.07 | 3.01 | 2.94 | 2.90 | 2.86 | 2.83 |
| **10** | 4.96 | 4.10 | 3.71 | 3.48 | 3.33 | 3.22 | 3.14 | 3.07 | 3.02 | 2.98 | 2.91 | 2.85 | 2.77 | 2.74 | 2.70 | 2.66 |
| **11** | 4.84 | 3.98 | 3.59 | 3.36 | 3.20 | 3.09 | 3.01 | 2.95 | 2.90 | 2.85 | 2.79 | 2.72 | 2.65 | 2.61 | 2.57 | 2.53 |
| **12** | 4.75 | 3.89 | 3.49 | 3.26 | 3.11 | 3.00 | 2.91 | 2.85 | 2.80 | 2.75 | 2.69 | 2.62 | 2.54 | 2.51 | 2.47 | 2.43 |
| **13** | 4.67 | 3.81 | 3.41 | 3.18 | 3.03 | 2.92 | 2.83 | 2.77 | 2.71 | 2.67 | 2.60 | 2.53 | 2.46 | 2.42 | 2.38 | 2.34 |
| **14** | 4.60 | 3.74 | 3.34 | 3.11 | 2.96 | 2.85 | 2.76 | 2.70 | 2.65 | 2.60 | 2.53 | 2.46 | 2.39 | 2.35 | 2.31 | 2.27 |
| **15** | 4.54 | 3.68 | 3.29 | 3.06 | 2.90 | 2.79 | 2.71 | 2.64 | 2.59 | 2.54 | 2.48 | 2.40 | 2.33 | 2.29 | 2.25 | 2.20 |
| **16** | 4.49 | 3.63 | 3.24 | 3.01 | 2.85 | 2.74 | 2.66 | 2.59 | 2.54 | 2.49 | 2.42 | 2.35 | 2.28 | 2.24 | 2.19 | 2.15 |
| **17** | 4.45 | 3.59 | 3.20 | 2.96 | 2.81 | 2.70 | 2.61 | 2.55 | 2.49 | 2.45 | 2.38 | 2.31 | 2.23 | 2.19 | 2.15 | 2.10 |
| **18** | 4.41 | 3.55 | 3.16 | 2.93 | 2.77 | 2.66 | 2.58 | 2.51 | 2.46 | 2.41 | 2.34 | 2.27 | 2.19 | 2.15 | 2.11 | 2.06 |
| **19** | 4.38 | 3.52 | 3.13 | 2.90 | 2.74 | 2.63 | 2.54 | 2.48 | 2.42 | 2.38 | 2.31 | 2.23 | 2.16 | 2.11 | 2.07 | 2.03 |
| **20** | 4.35 | 3.49 | 3.10 | 2.87 | 2.71 | 2.60 | 2.51 | 2.45 | 2.39 | 2.35 | 2.28 | 2.20 | 2.12 | 2.08 | 2.04 | 1.99 |
| **21** | 4.32 | 3.47 | 3.07 | 2.84 | 2.68 | 2.57 | 2.49 | 2.42 | 2.37 | 2.32 | 2.25 | 2.18 | 2.10 | 2.05 | 2.01 | 1.96 |
| **22** | 4.30 | 3.44 | 3.05 | 2.82 | 2.66 | 2.55 | 2.46 | 2.40 | 2.34 | 2.30 | 2.23 | 2.15 | 2.07 | 2.03 | 1.98 | 1.94 |
| **23** | 4.28 | 3.42 | 3.03 | 2.80 | 2.64 | 2.53 | 2.44 | 2.37 | 2.32 | 2.27 | 2.20 | 2.13 | 2.05 | 2.01 | 1.96 | 1.91 |
| **24** | 4.26 | 3.40 | 3.01 | 2.78 | 2.62 | 2.51 | 2.42 | 2.36 | 2.30 | 2.25 | 2.18 | 2.11 | 2.03 | 1.98 | 1.94 | 1.89 |
| **25** | 4.24 | 3.39 | 2.99 | 2.76 | 2.60 | 2.49 | 2.40 | 2.34 | 2.28 | 2.24 | 2.16 | 2.09 | 2.01 | 1.96 | 1.92 | 1.87 |
| **26** | 4.23 | 3.37 | 2.98 | 2.74 | 2.59 | 2.47 | 2.39 | 2.32 | 2.27 | 2.22 | 2.15 | 2.07 | 1.99 | 1.95 | 1.90 | 1.85 |
| **27** | 4.21 | 3.35 | 2.96 | 2.73 | 2.57 | 2.46 | 2.37 | 2.31 | 2.25 | 2.20 | 2.13 | 2.06 | 1.97 | 1.93 | 1.88 | 1.84 |
| **28** | 4.20 | 3.34 | 2.95 | 2.71 | 2.56 | 2.45 | 2.36 | 2.29 | 2.24 | 2.19 | 2.12 | 2.04 | 1.96 | 1.91 | 1.87 | 1.82 |