AFRICA UNIVERSITY
A United Methodist-Related Institution

"Investing in Africa's future"

COLLEGE BUSINESS PEACE LEADERSHIP AND GOVERNANCE

NMEC203: MATHEMATICS FOR ECONOMISTS

END-OF-SEMESTER FINAL EXAMINATIONS

NOVEMBER/ DECEMBER 2022

LECTURER: MR T. MASESE

DURATION: 3 HRS

INSTRUCTIONS

Answer **ANY FOUR QUESTIONS**

DO NOT repeat material.

Write legibly.

Credit is given to application of theory and practice

Question 1

a. Given the following supply and demand functions: $q_d = -5p + 100$; $q_s = 15p - 100$
   i. Find the equilibrium price and quantity **(6 marks)**
   ii. Find the inverse demand and supply functions, and verify that solving these simultaneously gives the same equilibrium price and quantity as in (i). **(5 marks)**
   iii. Illustrate (i) and (ii) graphically. **(5 marks)**

b. A supplier will only start to supply T-shirts when a price greater than $5 per unit is available. He or she will then increase output by 2 units (2 T-shirts) for every unit increase in price.
   i. Plot the supply function in the form Q= f (P). **(3 marks)**
   ii. Write down the equation of the supply function. **(3 marks)**
   iii. Find the value of Q when P = 15 from the graph. Confirm your answer from the equation **(3 marks)**

Question 2

Find the derivatives of each of the following using the power rule.

a. $y = 5x^7$

b. $Q = \dfrac{12}{L^{2.5}}$

(6 marks)

c. Determine the first, second and third derivatives of the following demand functions:

   i. $P = 25Q^4 - 10Q^2 + 200$

   ii. $TC = \dfrac{Q^3}{5} - 8Q^2 + \dfrac{5Q}{2} + 180$

(6 marks)

d. Differentiate each of the following, giving your answer with positive indices only:

i. $P = \dfrac{5Q + 2}{Q}$ **ii.** $y = 3\sqrt{Q^3}$

(6 marks)

e. Find dP / dQ and dQ / dP when $30P + 3Q - 81 = 0$. **(4 marks)**
f. Find dy / dx and dy/dx in terms of x, when $x^2 = 25y$ **(3 marks)**

Question 3

a. *Differentiate the following functions*

i. $y = (x2 + 1)^5$

ii. $y \bullet (x^2 1)(x^3 x^2)$

iii.

$$y = \frac{x^2 + 1}{x^3 + x^2}$$

(9 marks)

b. Assume that the total cost of a firm TC is given by $TC = 2q^2 + 5q + 1800$

i. Derive the average total cost function (ATC) for this firm **(3 marks)**

ii. Find the marginal cost (MC) function **(4 marks)**

iii. Find the output at which average cost is at its minimum **(3 marks)**

c. Assume that total cost is given by the linear function $TC = 2q + 20$. What is the Fixed Cost, Variable cost, Average variable cost and marginal cost for this firm **(6 marks)**

Question 4

a. The total cost function for Delta Corporation is given by $TC = 3q^2 + 5q + 48$,

i. Sketch the graph of the total cost function for q = 0 to $q = 5$. **(3 marks)**

ii. Find the marginal and average cost functions **(6 marks)**

iii. Show that AC is at its minimum when $q = 4$, and that $MC = AC$ at this output **(4 marks)**

iv. Sketch the graphs of the MC and AC functions, on the same axes. **(4 marks)**

b. Suppose we are told that the market demand function for a good is q $\dot{\circ}$-2p + 200

i. Write this demand curve as a function of p **(2 marks)**

ii. Plot the demand function in (i) **(2 marks)**

iii. If the total revenue function is TR ≡ pq, derive the total revenue function from the above demand curve **(2 marks)**

iv. Calculate the quantity at which revenue is maximized **(2 marks)**

Question 5

a. The supply function is given by P = 10 + 0.50Q

i. State and verbally describe the slope and intercepts (**3 marks**)

ii. Plot the supply function, P = 10 + 0.50 for 0 < Q < 100. (**3 marks**)

iii. Calculate P for Q=0, 20, 40, 60, 80, 100 (**5 marks**)

iv. Write the equation of the supply function in the form, P = h (Q), i.e. write P in terms of Q. Plot the graph of P in terms of Q **(3 marks)**

b. Suppose Malawi's economy is characterized by the following macroeconomic model:
$Y = E$
$E \cong C + I$
$C = 0.8Y + 100$
$I = 550$

where Y = aggregate income, E = aggregate expenditure, C = consumption by households and I = investment by firms.

i. Find the equilibrium levels of income and consumption, and illustrate diagrammatically. **(3 marks)**

ii. Suppose business leaders become more optimistic about the future demand for their products, and consequently increase their investment to 700. Find the new level of Y, the change in Y, the change in C, and the investment multiplier in this case **(3 marks)**

iii. Suppose the government, alarmed by the rise in income because it fears that inflation might increase, tries to persuade households to reduce their marginal propensity to consume in order to restore income to its previous level. Calculate the size of the reduction in the *MPC* required to achieve this. (Hint: you need to derive an equation in which 1- *MPC* is the dependent variable.) **(3 marks)**

END OF PAPER