"Investing in Africa's Future"

COLLEGE OF BUSINESS PEACE LEADERSHIP AND GOVERNANCE

NMMS 105: MATHEMATICS FOR BUSINESS II

END OF FIRST SEMESTER FINAL EXAMINATIONS

NOVEMBER 2022

LECTURER: MUGWAGWA T. M

TIME: 3 HOURS

INSTRUCTIONS

Answer ***all*** questions from Section A and ***any*** three from Section B

Marks allocated to each question are indicated in brackets at the end of the question.

Total Marks: 100.

Write legibly.

Section A (40 Marks)

Question One

A firm produces x tonnes of output at a total cost $C(x) = 0.1\ x^3 - 4x^2 + 20x + 5$ Find

(i) Average cost

(ii) Average Variable Cost

(iii) Average Fixed Cost

(iv) Marginal Cost

(v) Marginal Average Cost [10]

Question Two

The demand curve for a monopolist is given by $x = 100 - 4p$

(i) Find the total revenue, average revenue and marginal revenue.

(ii) At what value of x, the marginal revenue is equal to zero?

[8]

Question Three

Given the demand function $q= (1{,}200 - 2p)^{0.5}$, what is elasticity of demand when quantity is 30?

[7]

Question Four

a) Investigate the maxima and minima of the function $2x^3 + 3x^2 - 36x + 10$.

b) If the marginal revenue for a commodity is $MR = 9 - 6x^2 + 2x$, find the total revenue and demand function.

[8+7]

SECTION B (60 Marks)

Answer ***any*** three questions from this section.

Question Five

(a) A Transport Company has two types of trucks, Type A and Type B. Type A has a refrigerated capacity of 20 m^3 and a non-refrigerated capacity of 40 m^3 while Type B has the same overall volume with equal sections for refrigerated and non-refrigerated stock. A grocer needs to hire trucks for the transport of 3,000 m^3 of refrigerated stock and 4 000 m^3 of non-refrigerated stock. The cost per kilometre of a Type A is $30, and $40 for Type B. How many trucks of each type should the grocer rent to achieve the minimum total cost?

(b) A School is preparing a trip for 400 students. The company who is providing the transportation has 10 buses of 50 seats each and 8 buses of 40 seats, but only has 9 drivers available. The rental cost for a large bus is $800 and $600 for the small bus. Calculate how many buses of each type should be used for the trip for the least possible cost.

[10 +10]

Question Six

a) The marginal cost C(x) and marginal revenue R¢(x) are given by C(x) = 20 + 0.05x and R(x) = 30. The fixed cost is USD 200. Determine the maximum profit.

b) The demand and supply functions under pure competition are $P_d = 16 - x^2$ and $P_s = 2x^2 + 4$. Find the consumers' surplus and producers' surplus at the market equilibrium price.

[10 +10]

Question Seven

(a) Find the inverse of the matrix A = $\begin{bmatrix} 2 & 1 & 1 \\ 6 & 5 & -3 \\ 4 & -1 & 3 \end{bmatrix}$

(b) Hence or otherwise solve the equation

$$2x + y + z = 12$$
$$6x + 5y - 3z = 6$$
$$4x - y + 3z = 5$$

[10 + 10]

Question Eight

(a) Solve the differential equation $\frac{dy}{dt} = (1 - y)$.

(b) The rate at which an infection spreads in a poultry house is given by the equation

$\frac{dP}{dt} = 0.75(2500 - P)$, where t is time in days. If P = 0 at t = 0,

(i) Solve the differential equation to determine an expression for the number of poultry (P) infected at any time t.

(iii) Calculate the time taken for 1500 poultry to become infected. [20]

Question Nine

a) Determine the general solution of the difference equation $Y_{t+1} - 0.4Y_t = 0$. State whether the solution is stable or unstable.

b) The relationship between the number of whales inhabiting a given area to the numbers inhabiting that area during the previous seasons is given by the difference equation $P_n = 0.8P_{n-1} + 800$.

(a) Determine the general solution and discuss its stability

(b) If P0 = 5000 determine the particular solution. Graph the whale population for eight seasons [20]

END OF PAPER

Integration Techniques

Many integration formulas can be derived directly from their corresponding derivative formulas, while other integration problems require more work. Some that require more work are substitution and change of variables, integration by parts, trigonometric integrals, and trigonometric substitutions.

Basic formulas

Most of the following basic formulas directly follow the differentiation rules.

1. $\int kf(x)\,dx = k\int f(x)\,dx$
2. $\int [f(x) \pm g(x)]dx = \int f(x)\,dx \pm \int g(x)\,dx$
3. $\int k dx = kx + C$
4. $\int x^n\,dx = \frac{x^{n+1}}{n+1} + C, n \neq -1$
5. $\int \sin x\,dx = -\cos x + C$
6. $\int \cos x\,dx = \sin x + C$
7. $\int \sec^2 x\,dx = \tan x + C$
8. $\int \csc^2 x\,dx = -\cot x + C$
9. $\int \sec x \tan x\,dx = \sec x + C$
10. $\int \csc x \cot x\,dx = -\csc x + C$
11. $\int e^x\,dx = e^x + C$
12. $\int a^x\,dx = \frac{a^x}{\ln a} + C, a > 0, a \neq 1$
13. $\int \frac{dx}{x} = \ln|x| + C$
14. $\int \tan x\,dx = -\ln|\cos x| + C$
15. $\int \cot x\,dx = \ln|\sin x| + C$
16. $\int \sec x\,dx = \ln|\sec x + \tan x| + C$
17. $\int \csc x\,dx = -\ln|\csc x + \cot x| + C$
18. $\int \frac{dx}{\sqrt{a^2 - x^2}} = \arcsin\frac{x}{a} + C$
19. $\int \frac{dx}{a^2 + x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$
20. $\int \frac{dx}{x\sqrt{x^2 - a^2}} = \frac{1}{a}\,\mathrm{arcsec}\,\frac{x}{a} + C$

Applications of Integration

Producer Surplus = $p_o\, q_o$ - $\int g(x)dx$ with limits from 0 to q_0

Consumer Surplus =$\int (x)x$ - $p_o\, q_o$ with limits from 0 to q_0

Total Cost= $\int MC$dx

Total Revenue $= \int MR dx$

Differentiation

The chain rule

If y is a function of u, which is itself a function of x, then

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$$

differentiate the outer function and multiply by the derivative of the inner function

The product rule

If y = uv then $\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$

This rule tells you how to differentiate the product of two functions: multiply each function by the derivative of the other and add

The quotient rule

If $y = \frac{u}{v}$ then $\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

This rule tells you how to differentiate the quotient of two functions: bottom times derivative of top, minus top times derivative of bottom, all over bottom squared