"Investing in Africa's future"

COLLEGE OF BUSINESS PEACE LEADERSHIP AND GOVERNANCE

NCSC 101: DISCRETE STRUCTURES

END OF SECOND SEMESTER EXAMINATION

APRIL 2022

LECTURER: DR. WESTON GOVERE

DURATION: 3 HOURS

INSTRUCTIONS

Answer **ALL** questions in **Section A** and **ANY THREE** from **Section B.**

Total possible mark is **100.**

Start **each** question on a new page on your answer sheet.

The marks allocated to **each** question are shown at the end of the part question.

SECTION A (*Answer ALL Questions from this Section)*

A1 Define the following terms as used in Discrete Structures:

a) Inverse of a function.

b) Lemma.

c) Cardinality of a set.

d) Binary operation.

e) Commutative group.

[2, 2, 2, 2, 2]

A2.

a) Prove that if $A \subset B$ then $A \cap B = A$.
b) Prove that there is only one empty set.

[4, 4]

A3.

a) Using Boolean algebra show that $\neg(p \vee q) \vee \neg(p \vee \neg q)$ is equivalent to $\neg p$.
b) Find the conjunction of the propositions *p* and *q* where *p* is the proposition "Rebecca's PC has more than 16 GB free hard disk space" and *q* is the proposition "The processor in Rebecca's PC runs faster than 1 GHz."

c) Write each of these statements in the form "if *p*, then *q*" in English.
  i. It is necessary to walk 8 miles to get to the top of Long's Peak.
  ii. To get tenure as a professor, it is sufficient to be world famous.
  iii. Your guarantee is good only if you bought your CD player less than 90 days ago.

d) Explain, without using a truth table, why $(p \vee q \vee r) \wedge (\neg p \vee \neg q \vee \neg r)$ is true when at least one of *p*, *q*, and *r* is true and at least one is false, but is false when all three variables have the same truth value.

[4, 2, 3, 3]

A4.

a) Prove by mathematical induction that $5^n - 2^n$ is divisible by 3.

b) If x and y are non-negative reals prove by contradiction the claim that $\frac{x+y}{2} \geq \sqrt{xy}$

[5, 5]

SECTION B (*Answer any* ***THREE*** *Questions from this Section)*

B5

a) Let R be the relation with ordered pairs (Abdul, 22), (Brenda, 24), (Carla, 21), (Desire, 22), (Eddie, 24), and (Felicia, 22). Here each pair consists of a graduate student and this student's age. Specify a function determined by this relation.

b) Define a relation A on the set of real numbers R be defined as follows $\forall\ a\ , b \in R,\ a\, A\, b \Leftrightarrow a < b$

   i) Is A reflexive?

   ii) Is A symmetric?

   iii) Is A transitive?

c) Let R be a relation xRy if and only if $x - y$ is divisible by 5, $x, y \in \mathbb{N}$. Prove that R is an equivalence relation.

d) Let $R_1 = (A, A, E_1)$ and $R_2 = (A, A, E_2)$ be two relations within the set ,$A = \{2,3,4,6\}$, where $E_1 = \{(x, y): x$ is a multiple of $y\}$ and $E_2 = \{(x, y): x$ and y have no common factors $\}$

   i) Write E_1 and E_2 as two sets of ordered pairs.

   ii) Determine the domain and range of R_1.

   iii) Display the diagraph of R_1.

   iv) Determine the relation matrix of R_2.

[3, 3, 5, 2, 2, 3, 2]

B6

a) Let $f: A \to B$ be a function. Define the following terms:

   i) f is surjective

   ii) f is one-to-one

b) For each of the following functions $f: \mathbb{R} \to \mathbb{R}$ find the domain and range with justification and verify whether the function is surjective, injective or bijective.

   i) $f(x) = x + \cos x$

ii) $f(x) = x - 4$

iii) $f(x) = \frac{2}{(x-3)^2}$

c) Find $f \ o \ g$ and $g \ o \ f$ where $f(x) = \sqrt{x+1}$ and $g(x) = \frac{1}{x^2}$

[2, 2, 4, 4, 4, 4]

B7

a) How many possible outcomes are there when a fair coin is tossed three times?

b) In a survey of 200 people about whether they like apples (A), bananas (B), and oranges (C), the following information was obtained:

$|A| = 112, |B| = 89, \ |C| = 71, |A \cap B| = 32, |A \cap C| = 26,$
$|B \cap C| = 43, \quad |A \cap B \cap C| = 20$

i. How many people like exactly one of these fruits?
ii. How many people like none of this fruit?
iii. How many people do not like oranges?

c) Let $A = \{1,2,3\}, B = \{a, b\}$ and $C = \{m, n\}$. Find $A \times (B \times C)$

d) Let A, B, C and D be sets. Prove the following:

i. $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$.
ii. $(A \times B) \cap (C \times D) = (A \cap C) \times (B \cap D)$.

. **[3, 2, 2, 2, 3, 4, 4]**

B8

a) Determine whether the statement is a tautology, a contingency or a contradiction $p \rightarrow (q \rightarrow p)$

b) Use a truth table to show that $\neg(p \lor q) \lor \neg(p \lor \neg q)$ is equivalent to $\neg p$.

c) Show that $p \oplus q$ is equivalent to $(p \land \neg q) \lor (\neg p \land q)$ using equivalence laws.

d) Propositions p and q are defined on the universal set $U = \{quadrilaterals\}$. Let p be the proposition "x is a square" and q be the proposition "x is a rectangle". If the implication is given as $q \rightarrow p$, then state the converse, inverse and contrapositive in both symbolic form and in words.

e) Symbolize each of the following statements using quantifiers, predicates and logical connectives. Determine the truth value of the statement and negate it.

i) For every natural number x, there is another natural number y such that their sum is equal to 1.

ii) All mathematicians wear glasses.

(Hint: Define a predicate and the domain).

[3, 3, 4, 6, 2, 2]

END OF PAPER