"Investing in Africa's future"

COLLEGE OF BUSINESS PEACE LEADERSHIP AND GOVERNANCE (CBPLG)

DISCRETE STRUCTURES – CSC 102

END OF FIRST SEMESTER EXAMINATIONS

MAY/JUNE 2020

LECTURER: Mr. Weston Govere

DURATION: 3 HOURS

INSTRUCTIONS

Answer **ANY ONE** of the given questions. Each question carry **50** marks.

Show all working on your answer sheet.

The marks allocated to **each** question are shown at the end of the part question.

Question 1

a) Suppose that **Smartphone A** has **256 MB RAM** and **32 GB ROM**, and the resolution of its camera is **8 MP**; **Smartphone B** has **288 MB RAM** and **64 GB ROM**, and the resolution of its camera is **4 MP**; and **Smartphone C** has **128 MB RAM** and **32 GB ROM**, and the resolution of its camera is **5 MP**. Determine the truth value of each of these propositions.

i) Smartphone B has the most RAM of these three smartphones.

ii) Smartphone C has more ROM or a higher resolution camera than Smartphone B.

iii) Smartphone B has more RAM, more ROM, and a higher resolution camera than Smartphone A.

iv) If Smartphone B has more RAM and more ROM than Smartphone C, then it also has a higher resolution camera.

v) Smartphone A has more RAM than Smartphone B if and only if Smartphone B has more RAM than Smartphone A.

[5]

b) Find the conjunction of the propositions p and q where p is the proposition "Rebecca's PC has more than 16 GB free hard disk space" and q is the proposition "The processor in Rebecca's PC runs faster than 1 GHz." **[3]**

c) Determine whether the statement is a tautology, a contingency or a contradiction $p \to (q \to p)$ **[3]**

d)

i) Translate the following statement into symbolic form:
"If James does not quit his job then Mary will not get any money and James' family will be happy" **[2]**

ii) Let $P =$ I cheat; $Q =$ I will get caught;
$R =$ I will write an examination and $S =$ I will fail.
Translate $(R \wedge P) \to (Q \to S)$ into simple English **[2]**

e) Show that $p \oplus q$ is equivalent to $(p \wedge \neg q) \vee (\neg p \wedge q)$. **[4]**

f) Propositions p and q are defined on the universal set $U = \{quadrilaterals\}$. Let p be the proposition "x is a square" and q be the proposition "x is a rectangle". If the implication is given as $q \to p$, then state the converse, inverse and contrapositive in words. **[3]**

g) For each of these pairs of sets, determine whether the first is a subset of the second, the second is a subset of the first, or neither is a subset of the other.

   i) the set of airline flights from New York to New Delhi, **[2]**
   ii) the set of nonstop airline flights from New York to New Delhi **[2]**
   iii) the set of people who speak English, the set of people who speak Chinese **[2]**
   iv) the set of flying squirrels, the set of living creatures that can fly **[2]**

h)

   i) Prove that for any set A, $A \cup \emptyset = A$. **[3]**
   ii) For all sets A, B and C prove that if $A \subseteq B$ and $B \subseteq C$ then $A \subseteq C$. **[3]**
   iii) Let A, B and C be sets. Prove that
   $A \times (B \cap C) = (A \times B) \cap (A \times C)$ **[3]**

i) In a survey of 120 people, it was found out that: 65 read Daily News; 45 read Newsday; 42 read Herald; 20 read both Daily News and Newsday; 25 read both Daily News and Herald; 15 read both Newsday and Herald and 8 read all three newspapers. Find the number of people who read exactly one newspaper. **[4]**

j) Suppose that $A = \{a, b, c, d\}$ and $B = \{1, 2\}$, write down the following sets:

   iv) $A \times B$ **[2]**
   v) $P(B)$ **[2]**

k) Let U consist of all student residents in Varsity Hall, N the set of all student residents who attended netball practice on Thursday night, T the set of all student residents who watched TV on Thursday night and S the set of all student residents who studied maths on Thursday night.

   Express in terms of N, T, S, and the appropriate set operators:

i) The set of all student residents who did not attend netball practice on Thursday night. **[1]**

ii) The set of all student residents who, on Thursday night, studied maths but did not attend netball practice. **[2]**

Question 2

a) Determine whether the relation R on the set of all people is reflexive, symmetric, antisymmetric, and/or transitive, where $(a, b) \in R$ if and only if

i) a is taller than b. **[2]**

ii) a and b were born on the same day. **[2]**

iii) a has the same first name as b. **[2]**

iv) a and b have a common grandparent. **[2]**

b) Consider these relations on the set of integers:

i) $R_1 = \{(a, b) \mid a \leq b\}$,

ii) $R_2 = \{(a, b) \mid a > b\}$,

iii) $R_3 = \{(a, b) \mid a = b \text{ or } a = -b\}$,

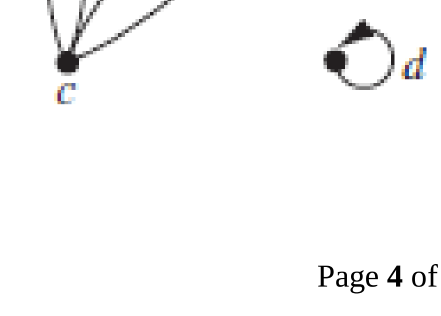
iv) $R = \{(a, b) \mid a = b\}$,

v) $R = \{(a, b) \mid a = b + 1\}$,

vi) $R = \{(a, b) \mid a + b \leq 3\}$.

Which of these relations contain each of the pairs $(1, 1), (1, 2), (2, 1), (1, -1)$, and $(2, 2)$? **[9]**

c) List the ordered pairs in the relations represented by the directed graphs.



i) **[3]**

ii) 

[2]

d) Let R be the relation with ordered pairs (Abdul, 22), (Brenda, 24), (Carla, 21), (Desire, 22), (Eddie, 24), and (Felicia, 22). Here each pair consists of a graduate student and this student's age. Specify a function determined by this relation. **[3]**

e) Show that

i) $f(x) = 5x + 2$ is surjective for all $x \in \mathbb{R}$. **[3]**

ii) the function $f: \mathbb{R} \to \mathbb{R}$ defined by $f(x) = \frac{(x+1)^3}{(x-1)^3}$ is bijective.

[4]

f)

i) Data stored on a computer disk or transmitted over a data network are usually represented as a string of bytes. Each byte is made up of 8 bits. How many bytes are required to encode 100 bits of data? **[3]**

ii) In asynchronous transfer mode (ATM) (a communications protocol used on backbone networks), data are organized into cells of 53 bytes. How many ATM cells can be transmitted in 1 minute over a connection that transmits data at the rate of 500 kilobits per second? **[4]**

g) Given $f(x) = \frac{2x+1}{2-3x}$ where $\in \mathbb{R}$; $x \neq k$.

iii) State the value of k **[1]**

iv) find $f^{-1}(x)$ and clearly state the domain. **[3]**

h) If $f(x) = 5x - 5 : \ x \in \mathbb{R}$ and $g(x) = \frac{1}{2x+1} : \ x \in \mathbb{R}; \ x \neq \frac{1}{2}$, show that $f \ o \ g \neq g \ o \ f$ **[3]**

i) Prove that if $f: A \to B, \ \ g: B \to C$ and $gof: A \to C$ is onto, then g is onto C. **[4]**

Question 3

a) Prove by an appropriate method that for all integers n:

$2 + 4 + 6 + \cdots + 2n = n(n+1)$ **[4]**

b) Show that at least four of any 22 days must fall on the same day of the week. **[3]**

c) Show that the statement "Every positive integer is the sum of the squares of two integers" is false. **[3]**

d) What is wrong with this famous supposed "proof" that 1 = 2?

"Proof:" We use these steps, where *a* and *b* are two equal positive integers.

Step	**Reason**
1. $a = b$	Given
2. $a^2 = ab$	Multiply both sides of (1) by a
3. $a^2 - b^2 = ab - b^2$	Subtract b^2 from both sides of (2)
4. $(a - b)(a + b) = b(a - b)$	Factor both sides of (3)
5. $a + b = b$	Divide both sides of (4) by $a - b$
6. $2b = b$	Replace a by b in (5) because $a = b$ and simplify
7. $2 = 1$	Divide both sides of (6) by b

[3]

e) For each of the following prove using both the direct proof and contrapositive proof:

i) If n is an even integer then $3n + 5$ is odd. **[3]**

ii) Let n be an integer. If $n^2 + 5$ is odd then n is even. **[3]**

f) Show that these statements about the integer n are equivalent:

p_1: n is even.

p_2: $n - 1$ is odd.

p_3: n^2 is even. **[4]**

g) What is the probability that Abby, Barry, and Sylvia win the first, second, and third prizes, respectively, in a drawing if 200 people enter a contest and

i) no one can win more than one prize. **[2]**

ii) winning more than one prize is allowed. **[3]**

h) When a car owner needs her car serviced, she phones one of the three garages A, B or C. Of her phone calls, 30% are to garage A, 10% to B and 60% to C. The percentages of occasions that when the garage is phoned they can take the car in on the day of phoning are 20% for A, 6% for B and 9% for C.

i) Draw a tree diagram to represent the above information. **[3]**

ii) Find the probability that the garage phoned will be able to take the car in on the day of phoning **[3]**

iii) Given that the car owner phones a garage and the garage can take in on that day, find the probability that she phoned garage B. **[3]**

i) Show that if *X* and *Y* are independent random variables on a sample space *S*, then *E(XY) = E(X)E(Y)*. **[5]**

j) If X is the r.v the random number on a number a biased die. The probability mass function (p.m.f) of X is as shown

X	1	2	3	4	5	6
$P(X = x)$	1/6	1/6	1/5	a	1/5	1/6

Find

iv) the value of a **[2]**

v) E(X) **[2]**

vi) E(X^2) **[2]**

vii) Var(X) **[2]**

END OF EXAMINATION PAPER