AFRICA UNIVERSITY

(A United Methodist-Related Institution)

"Investing in Africa's Future"

FACULTY OF MANAGEMENT AND ADMINISTRATION

COURSE TITLE: CSC 201 – DISCRETE MATHEMATICS

SEMESTER 1: FINAL EXAMINATION – NOVEMBER, 2014

LECTURER: MR T. MAKAMBWA

TIME: 3 HOURS

INSTRUCTIONS

Answer all questions from Section A and any three from Section B
Total possible mark is 100

Start **each** question on a new page in your answer Booklet.

The marks allocated to **each** question are shown at the end of the section.

Show all your workings.

Credit will be awarded for logical, systematic and neat presentations.

Section A(40 Marks)

Answer *all* questions from this section

Question One

Let a = 8316 and b = 10 920.

(*a*) Find d = gcd (a, b), the greatest common divisor of a and b.**[4]**
(*b*) Find integers m and n such that $d = ma + nb$. **[6]**
(*c*) Find lcm(a, b), the least common multiple of a and b. **[2]**

Question Two

Let $a = 2^3 \cdot 3^5 \cdot 5^4 \cdot 11^6 \cdot 17^3$ and $b = 2^5 \cdot 5^3 \cdot 7^2 \cdot 11^4 \cdot 13^2$. Find gcd (a, b) and lcm (a, b). **[5]**

Question Three

Which of the following are true?

- *(a)* $446 \equiv 278 \pmod{7}$,
- *(b)* $793 \equiv 682 \pmod{9}$,
- *(c)* $269 \equiv 413 \pmod{12}$,
- *(d)* $473 \equiv 369 \pmod{26}$,
- *(e)* $445 \equiv 536 \pmod{18}$
- *(f)* $383 \equiv 126 \pmod{15}$ **[12]**

Recall $a \equiv b \pmod{m}$ if and only if m divides $a - b$.

Question Four

Suppose $a = 5880$ and $b = 8316$.
(a) Express a and b as products of primes. **[4]**
(b) Find gcd (a, b) and lcm(a, b). **[5]**
(c) Verify that lcm$(a, b) = |ab| /$ gcd(a, b). **[2]**

Section B (60 Marks)

Answer *any* three questions in this section

Question Five

a) Prove that if n is even, then $(n + 1)^2$ is odd.
b) Prove that $\sqrt{2}$ is irrational.
c) Prove that if n^2 is odd, then n is odd using a proof by contradiction.
d) Prove that if n is an integer, then $n^3 - n$ is even

[5+5+5+5]

Question Six

a) Prove that $2m^2 + 3n^2 = 40$ has no solution in positive integers
b) Prove that $m^3 + 2n^2 = 36$
c) Prove that $2m^2 + 4n^2 - 1 = 2(m + n)$ has no solution in positive integers
d) Use induction to show that $5^n - 1$ is divisible by 4 for all $n \geq 1$

[5+5+5+5+5]

Question Seven

a) Using the 'quick method', prove that the following statement is a tautology: $(\sim p \wedge \sim q) \Rightarrow \sim (p \vee q)$ [8]

b) Write in predicate calculus notation using quantifiers and variables:
Some students cannot correctly answer all questions in this exam.

[4]

d) For the following sentence:
The statement that all statements are sentences but not all sentences are statements is false
i) Determine if it is a statement; **[4]**
(ii) If it is a statement, determine whether it is true or false, giving reasons for your answer. **[4]**

Question Eight

a) Using a direct proof, prove the following statement: **[4]**
"For any real number x, $20(x - 1) \leq 4x^2 + 5$"

b) Briefly explain how the following tautologies may be used in the method of proof by contradiction.
(i) $(\sim p \rightarrow (q \wedge \sim q)) \rightarrow p$
(ii) $(p \rightarrow q) \leftrightarrow (\sim q \rightarrow \sim p)$

[4]

c) Using proof by contradiction or otherwise, prove the following statements:

(i) "For all integers n, if n^3 is even then n is even". **[4]**
(ii) "There is no largest even integer" **[8]**

Question Nine

a) Let Ư = {*d*, *i*, *s*, *c*, *r*, *e*, *t*, *e*, *m*, *a*, *t*, *h*, *e*, *m*, *a*, *t*, *i*, *c*, *s*} be the Universal set. Let *S* = {*x* εƯ | *x* ε{*s*, *e*, *c*, *r*, *e*, *t*, *s*}}, *T* = {*x* ε 'Ư | *x* ε{*t*, *h*, *e*, *m*, *e*, *s*}} and

$C = \{x \in U \mid x \in \{t, a, c, t, i, c, s\}\}$ be subsets of the Universal set.
(i) Draw a Venn diagram showing S, T, C and U .**[6]**

ii) Write down the following sets:

a) $S \cup T$
b) $T - S$
c) S – C
d) C ∩T

[4]

b) Let $A = \{1\}$ and $B = \{4, 3, 2\}$. Write down the following sets:

(i) $(A \times B)$ **[4]**
(ii) $P(A \times B)$ **[6]**

END OF PAPER