AFRICA UNIVERSITY
A United Methodist-Related Institution

"Investing in Africa's Future"

COLLEGE OF BUSINESS PEACE LEADERSHIP AND GOVERNANCE

DISCRETE STRUCTURES-CSC 101

END OF SECOND SEMESTER EXAMINATIONS

APRIL/MAY 2018

LECTURER: Mr.Timothy Makambwa

DURATION: 3 HOURS

INSTRUCTIONS

Answer **ALL** the questions in **Section A** and any **Three questions from Section B** and each question has **20** marks. Total possible mark is **100.**

Start **each** question on a new page on your answer sheet.

The marks allocated to **each** question are shown at the end of the section.

Section A (40 Marks)

Answer *all* questions in this Section

Question One

Construct truth tables of the following:

a) $(P \rightarrow Q) \leftrightarrow (\neg P \,?\, Q)$
b) $P \rightarrow (Q \leftrightarrow P?Q)$
c) Prove that $P \rightarrow (Q?R) \equiv (P \rightarrow Q)?(P \rightarrow R)$
d) Prove that $(P?Q)?\neg(P?Q)$ is a tautology

[5+5+7+5]

Question Two

a) In the CSI Conference in Delhi, 500 delegates attended.200 of them could take tea, 350 could take coffee and 10 did not take either tea or coffee. Then answer the following questions:
   i. How many can take both tea and coffee,
   ii. How many can take tae only and
   iii. How many can take coffee only.

[2+2+2]

b) In a group of 191 students, 10 are taking English, Computer Science and Music, 36 are English and Computer Science, 20 are taking English and Music. 18 are taking Computer Science and Music, 65 are taking English, 76 Computer Science and 63 are taking Music. Then answer the followings:
   i. How many are taking English and Music but not Computer Science,
   ii. How many are taking Computer Science and Music but not English,
   iii. How many are taking Computer Science but neither English nor Music,
   iv. How many are taking none of the Three subjects

[3+3+3+3]

Section B

Answer any *three* in this Section

Question Three

Prove the following formulas for all positive integers n.

a) $1 + 2 + 3 + 4 + 5 + \ldots + n = n(n + 1) \div 2$

b) $2 + 4 + 6 + 8 + 10 + \ldots + 2n = n^2 + n$

c) $1 + 2 + 4 + 8 + 16 + \ldots + 2^{n-1} = 2^n - 1$

d) $1 + 3 + 9 + 27 + 81 + \ldots + 3^{n-1} = (3^n - 1) \div 2$

e) $1 + 4 + 9 + 16 + 25 + \ldots + n^2 = n(n + 1)(2n + 1) \div 6$

[4x5]

Question Four

Prove by induction for all positive integers n.

a) $2^{2n} - 1$ is a multiple of 3
b) 7 is a divisor of $2^{3n} - 1$
c) $n^3 + 2n$ is a multiple of 3
d) $n^5 - n \bmod 5 = 0$
e) $2^{n+2} + 3^{2n+1}$ is a multiple of 7

[4x5]

Question Five

Identify each proposition as a tautology, contradiction, or contingency/satisfiable .

a) $(p \wedge q) \rightarrow p$

b) $p \rightarrow (p \vee q)$

c) $p \rightarrow (p \rightarrow q)$

d) $p \rightarrow (q \rightarrow p)$

e) $\neg p \wedge \neg(p \rightarrow q)$

[4x5]

Question Six

Let $A = \{1, 2\}$ and $B = \{2, 3, 4, 5\}$. Write down the number of elements in each of the following sets:

a) $A \times A$

b) The set of functions from A to B

c) The set of one-to-one functions from A to B

d) The set of onto functions from A to B

e) The set of relations on B

f) The set of equivalence relations on B for which there are exactly two equivalence classes

g) The set of all equivalence relations on B

h) The set of one-to-one functions from B to A

i) The set of onto functions from B to A

j) The set of one-to-one and onto functions from B to B

[20]

Question Seven

Prove the following propositions.

a) $n < 2^2 \; \forall \; n \geq 1$

b) $2^n < n! \; \forall \; n \geq 4$

c) $3^n < n! \; \forall \; n \geq 7$

d) $2^n > n^n \; \forall \; n \geq 5$

e) $n! < n^n \; \forall \; n \geq 2$

[4x5]

Question Eight

Find the Greatest Common Divisor (GCD) of each pair using the Euclidean algorithm.

a) 275 and 115
b) 999 and 123
c) 456 and 144
d) 725 and 1000

[5X4]

End of Paper