AFRICA UNIVERSITY
A United Methodist-Related Institution

**COLLEGE OF BUSINESS PEACE LEADERSHIP AND GOVERNANCE**

**DISCRETE STRUCTURES-CSC 101**

**END OF SECOND SEMESTER EXAMINATIONS**

**November 2018**

**LECTURER: Mr.Timothy Makambwa**

**DURATION: 3 HOURS**

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## *INSTRUCTIONS*

Answer **ALL** the questions in **Section A** and any **Three questions from Section B** and each question has **20** marks. Total possible mark is **100.**

Start **each** question on a new page on your answer sheet.

The marks allocated to **each** question are shown at the end of the section.

**Section A (40 Marks)**

Answer *all* questions in this Section

**Question One**

Construct truth tables of the following:

a) $Q \wedge R \wedge \neg P$
b) $P \vee \neg Q \vee \neg R$
c) $(Q \wedge \neg P) \rightarrow R$
d) $\neg R \rightarrow (Q \wedge \neg P)$

[5+5+5+5]

**Question Two**

a) In a group of 100 persons, 72 people can speak English and 43 can speak French. How many can speak English only? How many can speak French only and how many can speak both English and French? [6]
b) In a competition, a school awarded medals in different categories. 36 medals in dance, 12 medals in dramatics and 18 medals in music. If these medals went to a total of 45 persons and only 4 persons got medals in all the three categories, how many received medals in exactly two of these categories? [14]

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**Section B**

Answer any *three* in this Section

**Question Three**

Prove the following formulas for all positive integers n.

a) $1 + 2 + 3 + 4 + 5 + \ldots + n = n(n + 1) \div 2$

b) $2 + 4 + 6 + 8 + 10 + \ldots + 2n = n^2 + n$

c) $1 + 2 + 4 + 8 + 16 + \ldots + 2^{n-1} = 2^n - 1$

d) $1 + 3 + 9 + 27 + 81 + \ldots + 3^{n-1} = (3^n - 1) \div 2$

e) $1 + 4 + 9 + 16 + 25 + \ldots + n^2 = n(n + 1)(2n + 1) \div 6$

[4x5]

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**Question Four**

Prove by induction for all positive integers n.

a) $2^{2n} - 1$ is a multiple of 3
b) 7 is a divisor of $2^{3n} - 1$
c) $n^3 + 2n$ is a multiple of 3
d) $n^5 - n \bmod 5 = 0$
e) $2^{n+2} + 3^{2n+1}$ is a multiple of 7

[4x5]

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**Question Five**

Identify each proposition as a tautology, contradiction, or contingency/satisfiable .

a) $(p \land q) \rightarrow p$

b) $p \rightarrow (p \lor q)$

c) $p \rightarrow (p \rightarrow q)$

d) $p \rightarrow (q \rightarrow p)$

e) $\neg p \land \neg(p \rightarrow q)$

[4x5]

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**Question Six**

Let $A = \{1, 2\}$ and $B = \{2, 3, 4, 5\}$. Write down the number of elements in each of the following sets:

a) $A \times A$

b) The set of functions from $A$ to $B$

c) The set of one-to-one functions from $A$ to $B$

d) The set of onto functions from *A* to *B*

e) The set of relations on *B*

f) The set of equivalence relations on *B* for which there are exactly two equivalence classes

g) The set of all equivalence relations on *B*

h) The set of one-to-one functions from *B* to *A*

i) The set of onto functions from *B* to *A*

j) The set of one-to-one and onto functions from *B* to *B*

[20]

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**Question Seven**

Prove the following propositions.

a) $n < 2^2 \ \forall \ n \geq 1$

b) $2^n < n! \ \forall \ n \geq 4$

c) $3^n < n! \ \forall \ n \geq 7$

d) $2^n > n^n \ \forall \ n \geq 5$

e) $n! < n^n \ \forall \ n \geq 2$

[4x5]

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**Question Eight**

Find the Greatest Common Divisor (GCD) of each pair using the Euclidean algorithm.

a) (24, 54)
b) (18, 42)
c) (244,354)
d) (2415, 3289)

e) (4278, 8602)

f) (406, 555)

[20]

**End of Paper**