AFRICA UNIVERSITY

A United Methodist-Related Institution

COLLEGE OF BUSINESS PEACE LEADERSHIP AND GOVERNANCE

NCSC 102: ALGEBRA

END OF SECOND SEMESTER EXAMINATIONS

NOVEMBER 2018

LECTURER: P TARAMBAWAMWE

DURATION: (3 HRS)

INSTRUCTIONS

ANSWER ALL QUESTIONS

SHOW ALL WORKING.

Q1a.

If the equations below can be represented as the matrix equation $AX=B$, where

$$x-2y-z=2$$
$$x+y=1$$
$$-2x+y-3z=-23$$

(i) What is the matrix A? **(2 marks)**

(ii) What is the matrix B? **(1 marks)**

(iii) Find the matrix A^{-1}. **(6 marks)**

(iv) Use your answers to the previous three parts of this question to find the values of x,y and z. **(5 marks)**

b. Solve for Y, Cd and M

$$\begin{bmatrix} 1 & -1 & 0 \\ 0 & -m & 1 \\ \frac{a}{1+m} & -1 & 0 \end{bmatrix} \begin{bmatrix} Y \\ C_d \\ M \end{bmatrix} = \begin{bmatrix} I+X \\ 0 \\ 0 \end{bmatrix}$$

(7 marks)

c. Find all solutions of the linear system

$$x1-2x2+x3-x4=0$$
$$2x1-3x2+4x3-3x4=0$$
$$3x1-5x2+5x3-4x4=0$$

(7 marks)

Q2

With respect to a fixed origin O, the lines l_1 and l_2 are given by the equations

$$l_1: \quad \mathbf{r} = \begin{pmatrix} 6 \\ -3 \\ -2 \end{pmatrix} + \lambda \begin{pmatrix} -1 \\ 2 \\ 3 \end{pmatrix}, \qquad l_2: \quad \mathbf{r} = \begin{pmatrix} -5 \\ 15 \\ 3 \end{pmatrix} + \mu \begin{pmatrix} 2 \\ -3 \\ 1 \end{pmatrix},$$

where λ and μ are scalar parameters.

(a) Show that l_1 and l_2 meet and find the position vector of their point of intersection A.

(6)

(b) Find, to the nearest 0.1°, the acute angle between l_1 and l_2.

(3)

The point B has position vector $\begin{pmatrix} 5 \\ -1 \\ 1 \end{pmatrix}$.

(c) Show that B lies on l_1.

(1)

(d) Find the shortest distance from B to the line l_2, giving your answer to 3 significant figures.

(4)

Q3

(i) Find the vector equation of the line L that passes through the two points A(1;-2; 1) and B(2; 3; 1). **(3 marks)**
(ii) A plane P passes through the three points C(2; 1;-3); D(4;-1; 2) and E(3; 0; 1). Obtain two independent vectors that are parallel to P and hence, or otherwise, show that P has the vector equation r:(1; 1; 0) = 3**(5 marks)**
(iii) Find the coordinates of the point of intersection of the line L and the plane P. **(5 marks)**
(iv) Derive the vector equation of the plane that contains the line L and is perpendicular to the plane P. **(5 marks)**