AFRICA UNIVERSITY
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COLLEGE OF BUSINESS, PEACE, LEADERSHIP, AND GOVERNANCE (CBPLG)

MATHEMATICS FOR BUSINESS 2 –NMMS 105

END OF FIRST SEMESTER EXAMINATIONS

NOVEMBER 2019

LECTURER: Mr.Timothy Makambwa

DURATION: 3 HOURS

INSTRUCTIONS

Answer **ALL** the questions in **Section A** and any **Three questions from Section B** and each question has **20** marks. Total possible mark is **100.**

Start **each** question on a new page on your answer sheet.

The marks allocated to **each** question are shown at the end of the section.

Section A (40 Marks)

Answer all questions in this Section

Question One

Differentiate

(a) $Y = (2x + 1)^{10}$

(b) $Y = (x^2 + 3x - 5)^3$

(c) $Y = x^2(x + 5)^3$

(d) $Y = x^5(4x + 5)^2$

(e) $Y = \dfrac{x^2}{x^2 + 4}$ [15]

Question Two

Integrate the following:

a) $\int 30x^4\, dx$
b) $\int (24 + 7.2x^{-2})\, dx$
c) $\int 0.5/x\, dx$
d) $\int (48x - 0.4e^{-1.4x})\, dx$
e) $\int (65 + 3^x)\, dx$

[10]

Question Three

a) Investigate the maxima and minima of the function $2x^3 + 3x^2 -- 36x + 10$.
b) If the marginal revenue for a commodity is MR = $9 - 6x^2 + 2x$, find the total revenue and demand function.

[8+7]

Section B (60 Marks)

Answer any three Questions from this Section

Question Four

a) The Total cost function : $C = x^2 + 6x + 39$, find the Average Cost function [3]
b) The demand function is : $p = x^2 - 24x + 117$
   i. Find an expression for the Total Revenue [3]
   ii. Find the output when Total Revenue is maximised [8]
   iii. Find the point elasticity of demand when the Total Revenue is maximised [6]

Question Five

a) Given the Marginal cost function: MC= 92-2x find the expression of the Total Cost function: TC when the fixed is 800. [4]
b) The Marginal Revenue function is: MR = 112-2x, find an expression of the Total Revenue function: TC [4]

c) From the responses of a)and b) find the expression of the Profit function :P [2]

d) Find the Total Revenue and Total Cost [10]

Question Six

a) Given the Marginal cost function: $MC= x^2-28x+211$find the expression of the Total Cost function: TC given x=0 when TC =10. [4]

b) The demand function is: P = 200-8x, find an expression of the Total Revenue function [4]

c) From the responses of a)and b) find the output at which profit is maximised [8]

d) Sketch the graphs of the Total Revenue and Total Cost functions [4]

Question Seven

a) Solve by matrix method or otherwise the equations

$x - 2y + 3z = 1$, $3x - y + 4z = 3$, $2x + y - 2z = -1$ [15]

b) Solve by Cramer's rule the equations $6x - 7y = 16$, $9x - 5y = 35$. [5]

Question Eight

a) The demand and supply law under a pure competition are given by $p_d = 23 - x^2$ and $p_s = 2x^2 - 4$. Find the consumers' surplus and producers' surplus at the market equilibrium price. [10]

b) Under pure competition the demand and supply laws for commodity and $p_d = 56 - x^2$ and $p_s = 8 + \frac{x^2}{3}$ Find the consumers' surplus and producers' surplus at the equilibrium [10]

Question Nine

a) Find the general solution of the difference equation $Y_{t+1} - 0.95Y_t = 1000$

b) Find the particular solution, given $Y_5 = 20950$.

c) Determine whether the system will stabilize and if so ,what the stable value is .Plot the time to stability for t = 0 to 10 in steps of one

d) Solve the difference equation $3Y_{t+1} + 2Y_t = 44(0.8)^t$ given $Y_0 = 900$.

e) Show that the solution stabilizes and plot the time path to stability.

[20]

Question Ten

a) Solve the differential equation $y^1 = 12e^{0.6t}$, given y=80 when t=0

[5]

b) Find the particular solution of $y^1 + 2y = 6$, given y=1 when x=0

[5]

c) Population in a developing country is growing continuously at an annual rate of 3%. If the population is now 4.5 million, what will it be in 15 years' time?

Hint : $dy/dt = ry$ and $y = Ae^{rt}$

[10]

END OF PAPER

Mathematical Formulae

Integration Techniques

Many integration formulas can be derived directly from their corresponding derivative formulas, while other integration problems require more work. Some that require more work are substitution and change of variables, integration by parts, trigonometric integrals, and trigonometric substitutions.

Basic formulas

Most of the following basic formulas directly follow the differentiation rules.

1. $\int kf(x)\,dx = k\int f(x)\,dx$
2. $\int [f(x) \pm g(x)]\,dx = \int f(x)\,dx \pm \int g(x)\,dx$
3. $\int k\,dx = kx + C$
4. $\int x^n\,dx = \frac{x^{n+1}}{n+1} + C, n \neq -1$
5. $\int \sin x\,dx = -\cos x + C$
6. $\int \cos x\,dx = \sin x + C$
7. $\int \sec^2 x\,dx = \tan x + C$
8. $\int \csc^2 x\,dx = -\cot x + C$
9. $\int \sec x \tan x\,dx = \sec x + C$
10. $\int \csc x \cot x\,dx = -\csc x + C$
11. $\int e^x\,dx = e^x + C$
12. $\int a^x\,dx = \frac{a^x}{\ln a} + C, a > 0, a \neq 1$
13. $\int \frac{dx}{x} = \ln|x| + C$
14. $\int \tan x\,dx = -\ln|\cos x| + C$
15. $\int \cot x\,dx = \ln|\sin x| + C$
16. $\int \sec x\,dx = \ln|\sec x + \tan x| + C$
17. $\int \csc x\,dx = -\ln|\csc x + \cot x| + C$
18. $\int \frac{dx}{\sqrt{a^2 - x^2}} = \arcsin\frac{x}{a} + C$
19. $\int \frac{dx}{a^2 + x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$

20. $\int \frac{dx}{x\sqrt{x^2 - a^2}} = \frac{1}{a} \operatorname{arcsec} \frac{x}{a} + C$

Applications of Integration

∴ Producer Surplus = $p_0q_0 - \int gx\,dx$ with limits from 0 to q_0

Consumer Surplus = $\int gx\,dx - p_0q_0$ with limits from 0 to q_0

Total Cost = $\int MCdx$

Total Revenue = $\int MRdx$

Differentiation

The chain rule

If *y* is a function of *u*, which is itself a function of *x*, then

$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$

differentiate the outer function and multiply by the derivative of the inner function

The product rule

If $y = uv$ then = $\frac{dy}{dx} = u.\frac{dv}{dx} \quad x \quad v.\frac{du}{dx}$

This rule tells you how to differentiate the product of two functions: **multiply each function by the derivative of the other and add**

The quotient rule

If $y = \frac{u}{v}$ then $\frac{dy}{dx} = \frac{v.du/dx - u.dv/dx}{v^2}$

This rule tells you how to differentiate the quotient of two functions: **bottom times derivative of top, minus top times derivative of bottom, all over bottom squared**