AFRICA UNIVERSITY
(A United Methodist-Related Institution)

"Investing in Africa's Future"

COLLEGE OF BUSINESS, PEACE, LEADERSHIP AND GOVERNANCE

Quantitative Analysis 1 (MMS202)

Final Examination - November 2019

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3 Hours

INSTRUCTIONS

Answer **All** questions.

Start **each** question on a new page in your answer booklet.

The marks allocated are shown at the end of **each** question.

Show all your workings where appropriate.

Credit will be given for logical, systematic and neat presentations.

1. An experiment was conducted to measure the effectiveness of a new procedure for pruning grapes. Each of 13 workers was assigned the task of pruning an acre of grapes. The productivity measured in worker-hours per acre is recorded for each person.

4.4 4.9 4.2 4.4 4.8 4.9 4.8 4.5 4.3 4.8 4.7 4.4 4.2

Determine the
a) Measures of central location. **[7 marks]**
b) Coefficient of variation and interpret. **[6 marks]**
c) Distribution of the data. **[3 marks]**

2. A student is studying the amount of money the CBPLG students spent on purchasing books. The student randomly selected the CBPLG students and asked them the amount they spent on buying books. The table below shows the data for the last semester.

Month	January	February	March	April
Expenditure on book	50	45	50	60

a) Construct the probability distribution and sketch the graph. **[4 marks]**
b) Construct the cumulative probability distribution. **[2 marks]**
c) Find the standard deviation of the data. **[5 marks]**

3. Suppose the following data for road resurfacing project in the recent past are available. The connection between costs and kilometres is that they are based on the same project.

Distance (in kilometres):	1	3	4	5	7
Cost (in thousands of dollars):	6	14	10	14	26

(a) Draw the scatter plot to examine the relationship between the two variables. Verify your results. **[8 marks]**
(b) What is the estimated average increase in cost for each additional kilometre? **[3 marks]**
(c) Sate four formal assumptions of regression analysis. **[4 marks]**

4. The mileage recorded for a sample of company vehicles during a given week yielded the following data:

13.8	16.4	15.0	13.2	14.4	12.5	14.9	15.7
14.6	15.8	14.0	14.7	13.6	14.8	15.2	14.4
16.8	12.6	13.8	17.6	16.3	11.9	15.4	16.5
14.6	17.3	14.2	14.7	13.5	15.3	14.0	13.5
16.1	14.5	13.5	14.2	15.0	15.6	14.5	12.8

(a) Construct a frequency distribution table **[5 marks]**
(b) Use the frequency distribution table you constructed in (a) to calculate:
(i) D_3 and P_{76}. **[8 marks]**
(ii) The mean and variance of the grouped data. **[7 marks]**

5. It has been claimed on the basis of census results that 87 percent of households in Tonnelle village now have exclusive use of a fixed bath or shower with hot water supply. In the recent housing survey of this area, 246 respondents out of the 300 interviewed reported this exclusive usage.

   a) At 5 percent level of significance, is the claim supported by the sample data? **[5 marks]**

   b) Explain the possible outcomes of hypothesis testing **[4 marks]**

6. A researcher hypothesizes that the average number of sports that colleges offer for males is greater than the average number of sports that colleges offer for females. A sample of the number of sports offered by colleges is shown.

Males					Females				
6	11	11	8	15	6	8	11	13	8
6	14	8	12	18	7	5	13	14	6
6	9	5	6	9	6	5	5	7	6
6	9	18	7	6	10	7	6	5	5
15	6	11	5	5	16	10	7	8	5
9	9	5	5	8	7	5	5	6	5
8	9	6	11	6	9	18	13	7	10
9	5	11	5	8	7	8	5	7	6
7	7	5	10	7	11	4	6	8	7
10	7	10	8	11	14	12	5	8	5

   a. At $\alpha = 0.10$, is there enough evidence to support the claim? Assume σ_1 and $\sigma_2 = 3.3$. **[5 marks]**

   b. Construct a 95% confidence interval for the data. Explain how the confidence interval can be used to test hypothesis. **[7 marks]**

7. Attempt the following questions:

   a) Twenty percent of the population are thought to be carriers of a certain disease, although they themselves may show no symptoms. If this is true, evaluate the following probabilities for a sample of five people selected at random from the population:

      i) that none are carriers **[2 marks]**

      ii) that all five are carriers **[2 marks]**

      iii) that fewer than two are carriers **[3 marks]**

   b) Items are packed into boxes of 1000 and each item has a probability of 0.001 of having some type of faulty.

      i) What is the probability that a box will contain fewer than three defective items? **[3 marks]**

      ii) If the company sells 100 000 boxes per year and guarantees fewer than three defectives per box, what is the expected number of guarantee claims? **[2 marks]**

iii) The replacement of a box returned under guarantee costs $150. What is the expected cost of guarantee claims? **[3 marks]**

iv) The boxes sell at $100 but cost $60 to produce and distribute. What is the company's expected profit for sales of boxes? **[2 marks]**

End of paper

ADDITIONAL INFORMATION

1. Sturge's Rule:

   Number of class, $C = 1 + 3.3\log n$

   Class width, $i > \dfrac{range}{C}$

2. Mean of grouped data $= \dfrac{\sum_{i=1}^{n} fx_i}{n}$

3. Mean of ungrouped data $= \dfrac{\sum_{i=1}^{n} x_i}{n}$

4. Mode $= L_{mo} + \left(\dfrac{\Delta_1}{\Delta_1 + \Delta_2}\right)i$

5 Median $= L_{me} + \left(\dfrac{\frac{n}{2} - F}{f_m}\right)i$

6. Standard deviation: $s = \sqrt{\dfrac{\sum_{i=1}^{n} fx_i^2 - \dfrac{\left(\sum_{n=1}^{n} fx_i\right)^2}{n}}{n-1}}$

7. Standard Deviation of ungrouped data: $s = \sqrt{\dfrac{\sum_{i=1}^{n} x_i^2 - \dfrac{\left(\sum_{i=1}^{n} x_i\right)^2}{n}}{n-1}}$

8. Coefficient of skewness: $S_k = \dfrac{3(mean - median)}{s} = \dfrac{mean - \text{mod}e}{s}$

9. Conditional probability: $P(A \setminus B) = \dfrac{P(A \cap B)}{P(A)}$

10. Binomial Distribution
    - $P(X = x) = nC_x p^x q^{n-x}$

11. Poisson Distribution
    - $P(X = x) = \dfrac{e^{-\lambda}\lambda^x}{x!}$

12. Hypothesis testing (single mean)
    - $Z = \dfrac{\bar{X} - \mu}{\frac{\sigma}{\sqrt{n}}}$
    - $t = \dfrac{\bar{X} - \mu}{\frac{s}{\sqrt{n}}}$, $df = n - 1$

13. Hypothesis testing (single proportion)
    - $Z = \dfrac{p - \pi}{\sqrt{\frac{\pi(1-\pi)}{n}}}$

14. Hypothesis testing (difference of two means)

- $Z = \frac{\bar{X}_1 - \bar{X}_2}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$
- $t = \frac{\bar{X}_1 - \bar{X}_2}{\sqrt{\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}}}$, *df* = smaller $(n_1 - 1;\ n_2 - 1)$
- $t = \frac{\bar{X}_1 - \bar{X}_2}{\sqrt{\frac{S_p^2}{n_1} + \frac{S_p^2}{n_2}}}$

  Where $S_p^2 = \frac{S_1^2(n_1) + S_2^2(n_2)}{n_1 + n_2 - 2}$, $df = n_1 + n_2 - 2$
- $t = \frac{\bar{D} - \mu_D}{\frac{S_D}{\sqrt{n}}}$, *df* = n – 1

15. Hypothesis testing (difference of two proportions)
    - $Z = \frac{p_1 - p_2}{\sqrt{\bar{p}\bar{q}\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$
16. Confidence Interval (Single mean)
    - $\bar{X} - Z_{\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}} \le \mu \le \bar{X} + Z_{\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}}$
    - $\bar{X} - t_{\frac{\alpha}{2}} \frac{S}{\sqrt{n}} \le \mu \le \bar{X} + t_{\frac{\alpha}{2}} \frac{S}{\sqrt{n}}$
17. Confidence Interval (Difference of two means)
    - $(\bar{X}_1 - \bar{X}_2) - Z_{\frac{\alpha}{2}} \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}} \le (\mu_1 - \mu_2) \le (\bar{X}_1 - \bar{X}_2) + Z_{\frac{\alpha}{2}} \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$
    - $(\bar{X}_1 - \bar{X}_2) - t_{\frac{\alpha}{2}} \sqrt{\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}} \le (\mu_1 - \mu_2) \le (\bar{X}_1 - \bar{X}_2) + t_{\frac{\alpha}{2}} \sqrt{\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}}$

      df = smaller $(n_1 - 1;\ n_2 - 1)$
    - $(\bar{X}_1 - \bar{X}_2) - t_{\frac{\alpha}{2}} \sqrt{\frac{S_p^2}{n_1} + \frac{S_p^2}{n_2}} \le (\mu_1 - \mu_2) \le (\bar{X}_1 - \bar{X}_2) + t_{\frac{\alpha}{2}} \sqrt{\frac{S_p^2}{n_1} + \frac{S_p^2}{n_2}}$

      Where $S_p^2 = \frac{S_1^2(n_1) + S_2^2(n_2)}{n_1 + n_2 - 2}$, $df = n_1 + n_2 - 2$
18. Confidence Interval (Single proportion)
    - $p - Z_{\frac{\alpha}{2}} \sqrt{\frac{pq}{n}} \le \pi \le p + Z_{\frac{\alpha}{2}} \sqrt{\frac{pq}{n}}$
19. Confidence Interval (Difference of two proportions)
    - $(p_1 - p_2) - Z_{\frac{\alpha}{2}} \sqrt{\frac{p_1 q_1}{n_1} + \frac{p_2 q_2}{n_2}} \le (\pi_1 - \mu_2) \le (p_1 - p_2) + Z_{\frac{\alpha}{2}} \sqrt{\frac{p_1 q_1}{n_1} + \frac{p_2 q_2}{n_2}}$
20. $Z = \frac{X - \mu}{\sigma}$
21. Weighted Mean: $\bar{X}_w = \frac{\sum xw}{\sum w}$
22. $P(B_i / C) = \frac{P(C / B_i) P(B_i)}{\sum_{i=1}^{n} P(C / B_i) P(B_i)}$,

23. $y = \beta_0 + \beta_1 x + e,$

$$\beta_1 = \frac{n\sum xy - \sum x \sum y}{n\sum y^2 - (\sum y)^2}$$

$$\beta_0 = \bar{y} + \beta_1 \bar{x}$$

$$r = \frac{n\sum xy - \sum x \sum y}{\{(n\sum x^2 - (\sum x)^2)(n\sum y^2 - (\sum y)^2)\}}$$

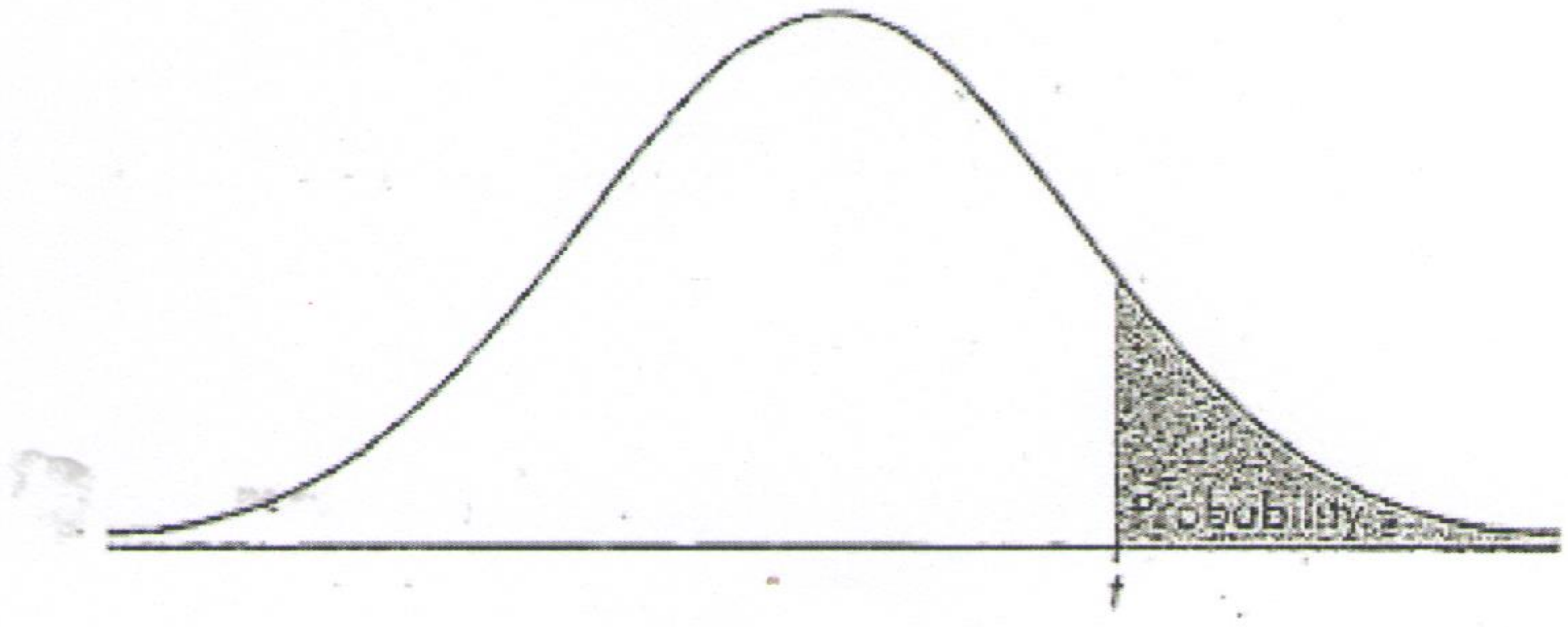


TABLE B: *t*-DISTRIBUTION CRITICAL VALUES

	Tail probability *p*											
df	.25	.20	.15	.10	.05	.025	.02	.01	.005	.0025	.001	.0005
1	1.000	1.376	1.963	3.078	6.314	12.71	15.89	31.82	63.66	127.3	318.3	636.6
2	.816	1.061	1.386	1.886	2.920	4.303	4.849	6.965	9.925	14.09	22.33	31.60
3	.765	.978	1.250	1.638	2.353	3.182	3.482	4.541	5.841	7.453	10.21	12.92
4	.741	.941	1.190	1.533	2.132	2.776	2.999	3.747	4.604	5.598	7.173	8.610
5	.727	.920	1.156	1.476	2.015	2.571	2.757	3.365	4.032	4.773	5.893	6.869
6	.718	.906	1.134	1.440	1.943	2.447	2.612	3.143	3.707	4.317	5.208	5.959
7	.711	.896	1.119	1.415	1.895	2.365	2.517	2.998	3.499	4.029	4.785	5.408
8	.706	.889	1.108	1.397	1.860	2.306	2.449	2.896	3.355	3.833	4.501	5.041
9	.703	.883	1.100	1.383	1.833	2.262	2.398	2.821	3.250	3.690	4.297	4.781
10	.700	.879	1.093	1.372	1.812	2.228	2.359	2.764	3.169	3.581	4.144	4.587
11	.697	.876	1.088	1.363	1.796	2.201	2.328	2.718	3.106	3.497	4.025	4.437
12	.695	.873	1.083	1.356	1.782	2.179	2.303	2.681	3.055	3.428	3.930	4.318
13	.694	.870	1.079	1.350	1.771	2.160	2.282	2.650	3.012	3.372	3.852	4.221
14	.692	.868	1.076	1.345	1.761	2.145	2.264	2.624	2.977	3.326	3.787	4.140
15	.691	.866	1.074	1.341	1.753	2.131	2.249	2.602	2.947	3.286	3.733	4.073
16	.690	.865	1.071	1.337	1.746	2.120	2.235	2.583	2.921	3.252	3.686	4.015
17	.689	.863	1.069	1.333	1.740	2.110	2.224	2.567	2.898	3.222	3.646	3.965
18	.688	.862	1.067	1.330	1.734	2.101	2.214	2.552	2.878	3.197	3.611	3.922
19	.688	.861	1.066	1.328	1.729	2.093	2.205	2.539	2.861	3.174	3.579	3.883
20	.687	.860	1.064	1.325	1.725	2.086	2.197	2.528	2.845	3.153	3.552	3.850
21	.686	.859	1.063	1.323	1.721	2.080	2.189	2.518	2.831	3.135	3.527	3.819
22	.686	.858	1.061	1.321	1.717	2.074	2.183	2.508	2.819	3.119	3.505	3.792
23	.685	.858	1.060	1.319	1.714	2.069	2.177	2.500	2.807	3.104	3.485	3.768
24	.685	.857	1.059	1.318	1.711	2.064	2.172	2.492	2.797	3.091	3.467	3.745
25	.684	.856	1.058	1.316	1.708	2.060	2.167	2.485	2.787	3.078	3.450	3.725
26	.684	.856	1.058	1.315	1.706	2.056	2.162	2.479	2.779	3.067	3.435	3.707
27	.684	.855	1.057	1.314	1.703	2.052	2.158	2.473	2.771	3.057	3.421	3.690
28	.683	.855	1.056	1.313	1.701	2.048	2.154	2.467	2.763	3.047	3.408	3.674
29	.683	.854	1.055	1.311	1.699	2.045	2.150	2.462	2.756	3.038	3.396	3.659
30	.683	.854	1.055	1.310	1.697	2.042	2.147	2.457	2.750	3.030	3.385	3.646
40	.681	.851	1.050	1.303	1.684	2.021	2.123	2.423	2.704	2.971	3.307	3.551
50	.679	.849	1.047	1.299	1.676	2.009	2.109	2.403	2.678	2.937	3.261	3.496
60	.679	.848	1.045	1.296	1.671	2.000	2.099	2.390	2.660	2.915	3.232	3.460
80	.678	.846	1.043	1.292	1.664	1.990	2.088	2.374	2.639	2.887	3.195	3.416
100	.677	.845	1.042	1.290	1.660	1.984	2.081	2.364	2.626	2.871	3.174	3.390
1000	.675	.842	1.037	1.282	1.646	1.962	2.056	2.330	2.581	2.813	3.098	3.300
∞	.674	.841	1.036	1.282	1.645	1.960	2.054	2.326	2.576	2.807	3.091	3.291
	50%	60%	70%	80%	90%	95%	96%	98%	99%	99.5%	99.8%	99.9%
	Confidence level *C*											

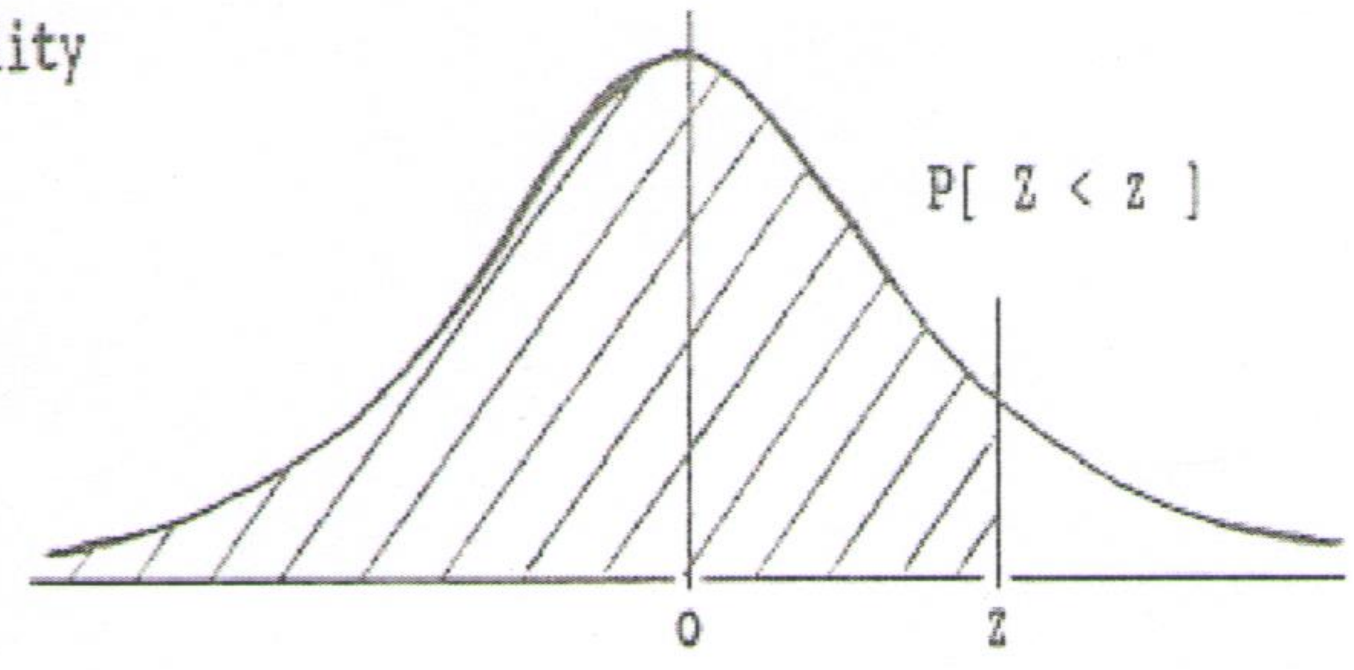
STANDARD STATISTICAL TABLES

1. Areas under the Normal Distribution

The table gives the cumulative probability up to the standardised normal value z

i.e.

$$P[\ Z < z\ ] = \int_{-\infty}^{z} \frac{1}{\sqrt{2\pi}} \exp(-\tfrac{1}{2}Z^2)\ dZ$$



z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5159	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7854
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8804	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9773	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9865	0.9868	0.9871	0.9874	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9924	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9980	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986

z	3.00	3.10	3.20	3.30	3.40	3.50	3.60	3.70	3.80	3.90
P	0.9986	0.9990	0.9993	0.9995	0.9997	0.9998	0.9998	0.9999	0.9999	1.0000