AFRICA UNIVERSITY

(A United Methodist-Related Institution)

"Investing in Africa's Future"

FACULTY OF MANAGEMENT AND ADMINISTRATION

COURSE TITLE: **MEC 203- MATHEMATICS FOR ECONOMISTS**

SEMESTER 1: **FINAL EXAMINATION NOVEMBER 2014**

LECTURER: **MR. L. NGENDAKUMANA**

TIME: **3 HOURS**

INSTRUCTIONS

Answer **ALL QUESTIONS in section A and any other TWO questions in section B.**
Total possible mark is **100.**

Start **each** question on a new page in your answer booklet.

The marks allocated to **each** question are shown at the end of the section.

Show all your workings.

Credit will be awarded for logical, systematic and neat presentations.

SECTION A

Question 1

A firm uses inputs K and L of capital and labor, respectively to produce a single output Q according to the production function $Q = F(K, L) = 120KL$. Suppose the prices of capital and labor are r and w, respectively and that the firm has m dollars to spend on the two inputs. The firm wishes to find the choice of inputs it can afford which maximizes output.

a) Formulate the output maximization problem **[2]**

b) Write down the Lagrangean function **[2]**

c) Find the optimal values of K(m) and L(m) that solve the maximization problem in (a) and the associated Lagrange multiplier $\lambda(m)$

[6]

d) Let $F^*(m) = 120K^*L^*$ be the value function Verify that $\frac{\partial F^*}{\partial m} = \lambda(m)$ **[3]**

Question 2

An economy has three industries- fishing, forestry and boat building. To produce one tone of fish requires the services of α fishing boats. To produce one ton of timber requires β tones of fish in order to feed the foresters. To produce one ton of fishing boat requires γ tons of timber. Suppose the final demands for the three goods are 85, 95, and 20 units, respectively. If x_1, x_2 and x_3 denote the number of units that have to be produced in the three sectors,

(a) Write down the Leontief model for the problem. **[1]**

(b) Find the number of units that has to be produced in each sector in order to meet the final demands. **[6]**

(c) What assumption would make sense for this economy to achieve efficient level of production of the three commodities? **[2]**

(d) Using a numerical illustration, briefly define and explain the Leontief model and show how the importance of this concept in Economics **[3]**

Question 3

a) A construction company has an order for 3 different types of houses: 5 of type A, 7 of type B, and 12 of type C. Write down 3 dimensional vector x whose coordinates give the number of houses for each type. Suppose that each house of type A requires 20 units of timber, type B requires 18 units, and type C requires 25 units. Write down a vector u that gives the different timber quantities required for each house of the three different types, A,B and C. Find the total timber requirement by computing the dot product u.x.

[5]

(b)Use Cramer's Rule to find Y and C when

Y = C+Io+Go C= a +bY

Where y is the national product and c is a private consumption. The symbols Io (private investment), Go (public consumption and investment), a and b all represent constants, with b< 1. (Actually, this is a typical case in which one should not use Cramer's rule, because Y and C can be found much more simply. How? **[10]**

SECTION B

Question 4

Using the simplest method of your choice, find solutions of the following problem:

Max (Min) f (x,y) $= x^2 + y^2$ subject to $g(x, y) = x^2 + xy + y^2 = 3$ **[25]**

Question 5

(i) The following set of equations describes behavior in the wheat market:

$$Q_t^d = 120 - 0.5P_t$$

$$Q_t^s = 30 + 0.3P_t$$

$$P_t = P_{t-1} - \alpha(Q_{t-1}^s - Q_{t-1}^d)$$

Where Q^d is quantity demanded, Q^s is quantity supplied, P is price and α is a positive parameter.

a. Solve for the long run equilibrium price and quantity **[5]**
b. Solve the first order difference equation in the price and find the particular solution if $P_0 = 200$ and α=1 **[10]**

(ii) Let Y_t , C_t and I_t denote the national product, consumption and investment respectively, in Zimbabwe at time t. Then at any time:

$$Y_t = C_t + I_t$$

Suppose furthermore that

$$C_t = 1000 + 0.7Y_{t-1}$$

and that $I_t = 500$ for all t.

(i) Deduce the difference equation for Y_t, and solve it for Yo= 2000 and t=2.

[10]

Question 6

Consider the following linked macroeconomic model of Zimbabwe and Kenya that trade with each other.

$$Yz = C_z + A_z + X_z - M_z$$

(i) $C_z = c_z Y_z$

$M_z = m_z Y_z$

$$Y_k = C_k + A_k + X_k - M_k$$

(i) $C_k = c_k Y_k$

$M_k = m_k Y_k$

Where: Y_z represents national income in Zimbabwe

Y_k represents national income in Kenya

A_z represents (exogenous) autonomous expenditure Zimbabwe

A_k represents (exogenous) autonomous expenditure Kenya

X_z represents exports in Zimbabwe

X_k represents exports in Kenya

M_z represents imports in Zimbabwe

M_k represents imports in Kenya

C_z represents consumption in Zimbabwe

C_k represents consumption in Kenya

(a) Interpret the two equations $X_z = M_k$ and $X_k = M_z$ **[5]**

(b) Given the equations in part (a), calculate the corresponding equilibrium values of Y_z and Y_k as functions of the exogenous variables. **[10]**

(c) How does an increase in A_z affect Y_k ? Interpret your answer. **[4]**

(d) Briefly explain the meaning of the various marginal propensities in the above model and provide their economic interpretation **[6]**

Question 7

(a) Consider the simple macroeconomic model described by the three equations:

Y= C + Ao, C= a+b(Y-T), T=d+tY

Where Y is income, is consumption, T is tax revenue, Ao is the constant (exogenous) autonomous expenditure, and a,b,d and t are all positive parameters. Find the equilibrium values of the endogenous variables Y, C and T by:

(i) successive elimination or substitution **5]**

(ii)Writing the equations in matrix form and applying Cramer's rule. **[8]**

(b) Let Y denote the DRC disposable income, C denote consumption, I investment and T denote tax (in millions of dollars). Suppose that the DRC economy model for the years 2005-2011 is described by the following equations:

(1) X=93.53
(2) C=0.712Y+95.05
(3) T=0.158 (C+X)-34.30
(4) Y=C+X-T

Solve for C, X, Y and T using any simplest method of your choice **[12]**

End of paper