“Investing in Africa’s Future”

FACULTY OF MANAGEMENT AND ADMINISTRATION

COURSE TITLE: MMS202 Quantitative Analysis 1

SEMESTER 1: Final Examination (Conventional) November 2014

LECTURER: Dr. S. Murairwa

TIME: 3 Hours

INSTRUCTIONS

Answer **All** questions in Section A and **any three** questions in Section B.

Start **each** question on a new page in your answer booklet.

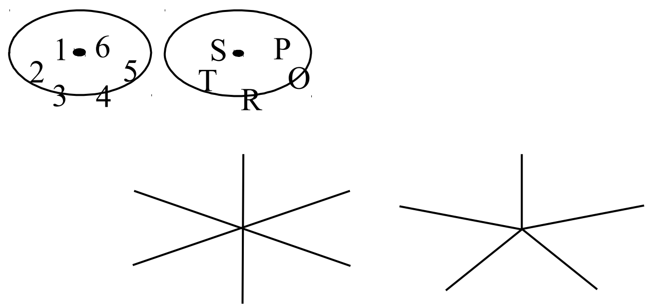
The marks allocated are shown at the end of **each** question.

Show all your workings.

Credit will be given for logical, systematic and neat presentations.

SECTION A: ANSWER ALL QUESTIONS

1. Suppose you spin each of the following two spinners:



   a) Find the sample space **[4 Marks]**
   b) What is the probability of spinning an even number and a vowel? **[4 Marks]**

2. There are 6 black pens and 8 blue pens in a jar. If you take a pen without looking and then take another pen without replacing the first pen,
   a) What is the probability that you will get 2 black pens? **[4 Marks]**
   b) Find the probability distribution of X, the number of blue pens in the selected sample. Plot the graph of the distribution of X **[6 Marks]**
   c) What is the expected value of the distribution in (b)? **[2 Marks]**

3. The bureau of Labour Statistics has sampled 30 communities in Zimbabwe and explained prices in each community at the beginning and end of August 2014 in order to find out approximately how the Consumer Price Index has change during the month. The percentage change in prices for the communities at the beginning of the month are given below:

0.8	0.2	-0.1	0.1	-0.2	0.2	0.3	0.5	-0.1	-0.2
0	0.6	0.3	0.2	1.0	-0.4	0	0.1	0.3	0.1
-0.5	-0.2	0	0.4	0.6	0	0.1	0.2	0.1	0.3

   a) Construct a frequency table for the data **[5 Marks]**
   b) Based on the frequency table you constructed in (a),
      i) sketch and explain the graph of the distribution of the data **[9 Marks]**
      ii) calculate and interpret the skewness to verify your answer in (i) **[6 Marks]**

SECTION B: ANSWER ANY THREE QUESTIONS

4. Attempt the following questions:

   a) Define the following Quantitative Analysis 1 terms:
      i) Open-ended question **[2 Marks]**
      ii) Pilot survey **[2 Marks]**
      iii) Snowball sampling **[2 Marks]**

iv) Panel survey **[2 Marks]**
v) Random sampling **[2 Marks]**

b) It has been claimed on the basis of census results that 87 percent of households in Tonnelle village now have exclusive use of a fixed bath or shower with hot water supply. In the recent housing survey of this area, 246 respondents out of the 300 interviewed reported this exclusive usage. At 5 percent level of significance, is the claim supported by the sample data? **[6 Marks]**

c) Explain the possible outcomes of hypothesis testing **[4 Marks]**

5. It has been decided to compare some of the results from the Arbour Housing survey (Survey 1) with those from the Pelouse Housing survey (Survey 2). Of particular interest was the level of monthly rent, a summary of which is given below:

Arbour housing survey	Pelouse Housing survey
Sample size = 180	Sample size = 150
Sample mean = $221.25	Sample mean = $206. 38
Sample variance = $89.48	Sample variance = $69.88

There is a significant difference between the two surveys.

a) Use the three methods of testing hypothesis to test the claim at 5 percent level of significance. Comment your results **[14 Marks]**

b) With a diagram and examples, explain the relationship between statistic and parameter **[6 Marks]**

6. A company has produced the following table to describe the travel of its employees

Reported cost of	Type of travel		
travel	Local	Commuter	Long distance
Under $1	60	20	0
$1 but under $5	87	46	17
Over $5	12	13	53

Use the measures of location and measures of dispersion to describe and contrast the various types of travel **[20 Marks]**

7. Attempt the following questions:

a) Twenty percent of the population are thought to be carriers of a certain disease, although they themselves may show no symptoms. If this is true, evaluate the following probabilities for a sample of five people selected at random from the population:
i) that none are carriers **[2 Marks]**
ii) that all five are carriers **[2 Marks]**
iii) that fewer than two are carriers **[3 Marks]**

b) Items are packed into boxes of 1000 and each item has a probability of 0.001 of having some type of faulty.

i) What is the probability that a box will contain fewer than three defective items? [**3 Marks**]

ii) If the company sells 100 000 boxes per year and guarantees fewer than three defectives per box, what is the expected number of guarantee claims? [**2 Marks**]

iii) The replacement of a box returned under guarantee costs $150. What is the expected cost of guarantee claims? [**3 Marks**]

iv) The boxes sell at $100 but cost $60 to produce and distribute. What is the company's expected profit for sales of boxes? [**2 Marks**]

c) A normal distribution has a mean of 30 and a standard deviation of 5; find the Z values equivalent to the X values given below:

i) 35 [**1 Mark**]

ii) 27 [**1 Mark**]

iii) 22.3 [**1 Mark**]

End of paper

ADDITIONAL INFORMATION

1. Sturge's Rule:

Number of class,
$$C = 1 + 3.3 \log n$$

Class width,
$$i > \frac{range}{C}$$

2. Mean of grouped data

$$= \frac{\sum_{i=1}^{n} fx_i}{n}$$

3. Mean of ungrouped data =

$$\frac{\sum_{i=1}^{n} x_i}{n}$$

4. Mode =

$$L_{mo} + \left(\frac{\Delta_1}{\Delta_1 + \Delta_2} \right) i$$

5 Median =

$$L_{me} + \left(\frac{\frac{n}{2} - F}{f_m} \right) i$$

6. Standard deviation:

$$S = \sqrt{\frac{\sum_{i=1}^{n} fx_i^2 - \frac{\left(\sum_{n=1}^{n} fx_i \right)^2}{n}}{n-1}}$$

7. Standard Deviation of ungrouped data:

$$S = \sqrt{\frac{\sum_{i=1}^{n} x_i^2 - \frac{\left(\sum_{i=1}^{n} x_i \right)^2}{n}}{n-1}}$$

8. Coefficient of skewness:

$$S_k = \frac{3(mean - median)}{s} = \frac{mean - \text{mod}\, e}{s}$$

9. Conditional probability:

$$P(A \setminus B) = \frac{P(A \cap B)}{P(A)}$$

10. Binomial Distribution

- $P(X=x) = nC_x p^x q^{n-x}$

11. Poisson Distribution

- $P(X=x) = \frac{e^{-\lambda} \lambda^x}{x!}$

12. Hypothesis testing (single mean)

- $Z = \frac{\acute{X} - \mu}{\frac{\sigma}{\sqrt{n}}}$
- $t = \frac{\acute{X} - \mu}{\frac{s}{\sqrt{n}}}$, *df* = n – 1

13. Hypothesis testing (single proportion)

- $Z = \frac{p - \pi}{\sqrt{\frac{\pi(1-\pi)}{n}}}$

14. Hypothesis testing (difference of two means)

- $Z = \frac{\acute{X}_1 - \acute{X}_2}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$
- $t = \frac{\acute{X}_1 - \acute{X}_2}{\sqrt{\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}}}$, *df* = smaller $(n_1 - 1; n_2 - 1)$
- $t = \frac{\acute{X}_1 - \acute{X}_2}{\sqrt{\frac{S_p^2}{n_1} + \frac{S_p^2}{n_2}}}$

  Where $S_p^2 = \frac{S_1^2(n_1) + S_2^2(n_2)}{n_1 + n_2 - 2}$, *df* = $n_1 + n_2 - 2$
- $t = \frac{\acute{D} - \mu_D}{\frac{S_D}{\sqrt{n}}}$, *df* = n – 1

15. Hypothesis testing (difference of two proportions)

- $Z = \frac{p_1 - p_2}{\sqrt{\acute{p}\acute{q}\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$

16. Confidence Interval (Single mean)

- $\acute{X} - Z_{\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}} \le \mu \le \acute{X} + Z_{\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}}$
- $\acute{X} - t_{\frac{\alpha}{2}} \frac{S}{\sqrt{n}} \le \mu \le \acute{X} + t_{\frac{\alpha}{2}} \frac{S}{\sqrt{n}}$

17. Confidence Interval (Difference of two means)

- $(\acute{X}_1 - \acute{X}_2) - Z_{\frac{\alpha}{2}} \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}} \le (\mu_1 - \mu_2) \le (\acute{X}_1 - \acute{X}_2) + Z_{\frac{\alpha}{2}} \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$
- $(\acute{X}_1 - \acute{X}_2) - t_{\frac{\alpha}{2}} \sqrt{\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}} \le (\mu_1 - \mu_2) \le (\acute{X}_1 - \acute{X}_2) + t_{\frac{\alpha}{2}} \sqrt{\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}}$

$df =$ smaller $(n_1 - 1; n_2 - 1)$

- $$(\acute{X}_1 - \acute{X}_2) - t_{\frac{\alpha}{2}} \sqrt{\frac{S_p^2}{n_1} + \frac{S_p^2}{n_2}} \leq (\mu_1 - \mu_2) \leq (\acute{X}_1 - \acute{X}_2) + t_{\frac{\alpha}{2}} \sqrt{\frac{S_p^2}{n_1} + \frac{S_p^2}{n_2}}$$

Where $S_p^2 = \frac{S_1^2(n_1) + S_2^2(n_2)}{n_1 + n_2 - 2}$, $df =$ $n_1 + n_2 - 2$

18. Confidence Interval (Single proportion)

- $$p - Z_{\frac{\alpha}{2}} \sqrt{\frac{pq}{n}} \leq \pi \leq p + Z_{\frac{\alpha}{2}} \sqrt{\frac{pq}{n}}$$

19. Confidence Interval (Difference of two proportions)

- $$(p_1 - p_2) - Z_{\frac{\alpha}{2}} \sqrt{\frac{p_1 q_1}{n_1} + \frac{p_2 q_2}{n_2}} \leq (\pi_1 - \mu_2) \leq (p_1 - p_2) + Z_{\frac{\alpha}{2}} \sqrt{\frac{p_1 q_1}{n_1} + \frac{p_2 q_2}{n_2}}$$

20. $Z = \frac{X - \mu}{\sigma}$

21. Weighted Mean: $\acute{X}_w = \frac{\sum xw}{\sum w}$

22.

$$P(B_i / C) = \frac{P(C / B_i) P(B_i)}{\sum_{i=1}^{n} P(C / B_i) P(B_i)}$$

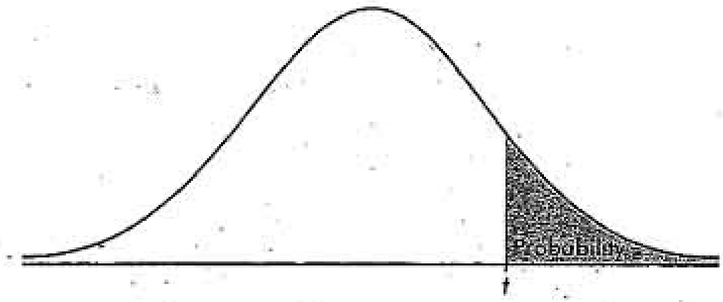


TABLE B: *t*-DISTRIBUTION CRITICAL VALUES

	Tail probability *p*											
df	.25	.20	.15	.10	.05	.025	.02	.01	.005	.0025	.001	.0005
1	1.000	1.376	1.963	3.078	6.314	12.71	15.89	31.82	63.66	127.3	318.3	636.6
2	.816	1.061	1.386	1.886	2.920	4.303	4.849	6.965	9.925	14.09	22.33	31.60
3	.765	.978	1.250	1.638	2.353	3.182	3.482	4.541	5.841	7.453	10.21	12.92
4	.741	.941	1.190	1.533	2.132	2.776	2.999	3.747	4.604	5.598	7.173	8.610
5	.727	.920	1.156	1.476	2.015	2.571	2.757	3.365	4.032	4.773	5.893	6.869
6	.718	.906	1.134	1.440	1.943	2.447	2.612	3.143	3.707	4.317	5.208	5.959
7	.711	.896	1.119	1.415	1.895	2.365	2.517	2.998	3.499	4.029	4.785	5.408
8	.706	.889	1.108	1.397	1.860	2.306	2.449	2.896	3.355	3.833	4.501	5.041
9	.703	.883	1.100	1.383	1.833	2.262	2.398	2.821	3.250	3.690	4.297	4.781
10	.700	.879	1.093	1.372	1.812	2.228	2.359	2.764	3.169	3.581	4.144	4.587
11	.697	.876	1.088	1.363	1.796	2.201	2.328	2.718	3.106	3.497	4.025	4.437
12	.695	.873	1.083	1.356	1.782	2.179	2.303	2.681	3.055	3.428	3.930	4.318
13	.694	.870	1.079	1.350	1.771	2.160	2.282	2.650	3.012	3.372	3.852	4.221
14	.692	.868	1.076	1.345	1.761	2.145	2.264	2.624	2.977	3.326	3.787	4.140
15	.691	.866	1.074	1.341	1.753	2.131	2.249	2.602	2.947	3.286	3.733	4.073
16	.690	.865	1.071	1.337	1.746	2.120	2.235	2.583	2.921	3.252	3.686	4.015
17	.689	.863	1.069	1.333	1.740	2.110	2.224	2.567	2.898	3.222	3.646	3.965
18	.688	.862	1.067	1.330	1.734	2.101	2.214	2.552	2.878	3.197	3.611	3.922
19	.688	.861	1.066	1.328	1.729	2.093	2.205	2.539	2.861	3.174	3.579	3.883
20	.687	.860	1.064	1.325	1.725	2.086	2.197	2.528	2.845	3.153	3.552	3.850
21	.686	.859	1.063	1.323	1.721	2.080	2.189	2.518	2.831	3.135	3.527	3.819
22	.686	.858	1.061	1.321	1.717	2.074	2.183	2.508	2.819	3.119	3.505	3.792
23	.685	.858	1.060	1.319	1.714	2.069	2.177	2.500	2.807	3.104	3.485	3.768
24	.685	.857	1.059	1.318	1.711	2.064	2.172	2.492	2.797	3.091	3.467	3.745
25	.684	.856	1.058	1.316	1.708	2.060	2.167	2.485	2.787	3.078	3.450	3.725
26	.684	.856	1.058	1.315	1.706	2.056	2.162	2.479	2.779	3.067	3.435	3.707
27	.684	.855	1.057	1.314	1.703	2.052	2.158	2.473	2.771	3.057	3.421	3.690
28	.683	.855	1.056	1.313	1.701	2.048	2.154	2.467	2.763	3.047	3.408	3.674
29	.683	.854	1.055	1.311	1.699	2.045	2.150	2.462	2.756	3.038	3.396	3.659
30	.683	.854	1.055	1.310	1.697	2.042	2.147	2.457	2.750	3.030	3.385	3.646
40	.681	.851	1.050	1.303	1.684	2.021	2.123	2.423	2.704	2.971	3.307	3.551
50	.679	.849	1.047	1.299	1.676	2.009	2.109	2.403	2.678	2.937	3.261	3.496
60	.679	.848	1.045	1.296	1.671	2.000	2.099	2.390	2.660	2.915	3.232	3.460
80	.678	.846	1.043	1.292	1.664	1.990	2.088	2.374	2.639	2.887	3.195	3.416
100	.677	.845	1.042	1.290	1.660	1.984	2.081	2.364	2.626	2.871	3.174	3.390
1000	.675	.842	1.037	1.282	1.646	1.962	2.056	2.330	2.581	2.813	3.098	3.300
∞	.674	.841	1.036	1.282	1.645	1.960	2.054	2.326	2.576	2.807	3.091	3.291
	50%	60%	70%	80%	90%	95%	96%	98%	99%	99.5%	99.8%	99.9%
	Confidence level *C*											

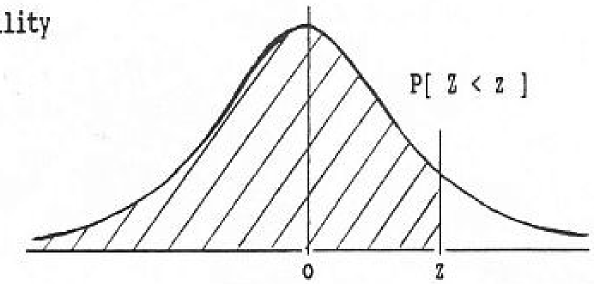
STANDARD STATISTICAL TABLES

1. Areas under the Normal Distribution

The table gives the cumulative probability up to the standardised normal value z

i.e.

$$P[\, Z < z \,] = \int_{-\infty}^{z} \frac{1}{\sqrt{2\pi}} \exp(-\tfrac{1}{2}Z^2)\, dZ$$



z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5159	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7854
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8804	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9773	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9865	0.9868	0.9871	0.9874	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9924	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9980	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986

z	3.00	3.10	3.20	3.30	3.40	3.50	3.60	3.70	3.80	3.90
P	0.9986	0.9990	0.9993	0.9995	0.9997	0.9998	0.9998	0.9999	0.9999	1.0000