AFRICA UNIVERSITY

(A United Methodist-Related Institution)

"Investing in Africa's Future"

FACULTY OF MANAGEMENT AND ADMINISTRATION

COURSE TITLE: **MEC 203- MATHEMATICS FOR ECONOMISTS**

SEMESTER 1: **FINAL EXAMINATION NOVEMBER 2013**

LECTURER: **MR. L. NGENDAKUMANA**

TIME: **3 HOURS**

INSTRUCTIONS

Answer **both questions in section A and two questions in section B.** Total possible mark is **60.**

Start **each** question on a new page in your answer booklet.

The marks allocated to **each** question are shown at the end of the section.

Show all your workings.

Credit will be awarded for logical, systematic and neat presentations.

SECTION A

Question 1

(i) The following set of equations describes behavior in the wheat market:

$$Q_t^d = 120 - 0.5P_t$$

$$Q_t^s = 30 + 0.3P_t$$

$$P_t = P_{t-1} - \alpha(Q_{t-1}^s - Q_{t-1}^d)$$

Where Q^d is quantity demanded, Q^s is quantity supplied, P is price and α is a positive parameter.

a. Solve for the long run equilibrium price and quantity **[3]**

b. Solve the first order difference equation in the price and find the particular solution if $P_0 = 200$ and α=1 **[7]**

(ii) Let Y_t , C_t and I_t denote the national product, consumption and investment respectively, in Zimbabwe at time t. Then at any time:

$$Y_t = C_t + I_t$$

Suppose furthermore that

$$C_t = 1000 + 0.7Y_{t-1}$$

and that $I_t = 500$ for all t.

(i) Deduce the difference equation for Y_t, and solve it for Yo= 2000 and t=2.

[5]

Question 2

An individual has m dollars to spend on three commodities X, Y, and Z. The utility enjoyed by consuming X, Y and Z units is given by

v(x, y, z)= x^2y^3z so he is faced with the problem:

Max v(x, y, z) subject to $x + y + z = m$

a. Write down the Lagrange function **[2]**

b. Find the optimal values of x(m), y(m), and z(m) that solve the maximization problem above and the associated Lagrange multiplier λ(m). **[7]**

c. Let $f^*(m) = x(m)^2 y(m)^3 z(m)$ be the value function. Verify that $\frac{df^*(m)}{dm} = \lambda(m)$

[3]

d. Using the concept of optimal value function, highlight the economic interpretation of the Lagrange multiplier **[3]**

SECTION B

Question 3

(i) (a) Given the matrix below:

$$A = \begin{pmatrix} a & a^2-1 & -3 \\ a+1 & 2 & a^2+4 \\ -3 & 4a & -1 \end{pmatrix}$$

For what values of a is the matrix A symmetric? Compute the matrix A by giving its appropriate value (s). **[3]**

(b) Show that the 2x 2 matrix $\begin{pmatrix} p & -q \\ q & p \end{pmatrix}$ is orthogonal

[2]

(ii) Consider the following linked macroeconomic model of Uganda and Zambia that trade with each other.

(i)
$$Y_U = C_U + A_U + X_U - M_U$$
$$C_U = c_U Y_U$$
$$M_U = m_U Y_U$$

(ii)
$$Yz = C_z + A_z + X_z - M_z$$
$$C_z = c_z Y_z$$
$$M_z = m_z Y_z$$

Where: Y_z represents national income in Zambia

Y_U represents national income in Uganda

A_z represents (exogenous) autonomous expenditure Zambia

A_U represents (exogenous) autonomous expenditure Uganda

X_z represents exports in Zambia
X_U represents exports in Uganda
M_z represents imports in Zambia
M_U represents imports in Uganda
C_z represents consumption in Zambia
C_U represents consumption in Uganda

(a) Interpret the two equations $X_z = M_U$ and $X_U = M_z$ **[2]**
(b) Given the equations in part (a), calculate the corresponding equilibrium values of Y_z and Y_U as functions of the exogenous variables. **[7]**
(c) How does an increase in A_z affect Y_U? Interpret your answer. **[1]**

Question 4

(i) An economy has three industries- fishing, forestry and boat building. To produce one tone of fish requires the services of α fishing boats. To produce one ton of timber requires β tones of fish in order to feed the foresters. To produce one ton of fishing boat requires γ tons of timber. Suppose the final demands for the three goods are 85, 95, and 20 units, respectively. If x_1, x_2 and x_3 denote the number of units that have to be produced in the three sectors,
(a) Write down the Leontief model for the problem. **[2]**
(b) Find the number of units that has to be produced in each sector in order to meet the final demands. **[6]**
(c) What assumption would make sense for this economy to achieve efficient level of production of the three commodities? **[2]**

(ii) Use Cramer's Rule to find Y and C when
$Y = C+Io+Go$ $\qquad$ $C= a +bY$

Where Y is the national product and C is a private consumption. The symbols Io (private investment), Go (public consumption and investment), a and b all represent constants, with b< 1. (Actually, this is a typical case in which one should not use Cramer's rule, because Y and C can be found much more simply. How? **[5]**

Question 5

(i) Let Y denote Angola disposable income, C denote consumption, I investment and T denote tax. Suppose that the Angolan economy model for the years 1970-1980 is described by the following equations:
(1) X=93.53
(2) C=0.712Y+95.05

(3) $T=0.158\ (C+X)-34.30$

(4) $Y=C+X-T$

Solve for C, X, Y and T using two different methods of your choice **[10]**

(ii) After defining both types of vectors, explain how dots vectors are computed. Use concrete examples to support your arguments **[5]**

……………………………………………..End of paper………………………………