

FACULTY OF MANAGEMENT AND ADMINISTRATION

- COURSE TITLE: MEC 203- MATHEMATICS FOR ECONOMISTS
- SEMESTER 1: FINAL EXAMINATION NOVEMBER 2013
- LECTURER: MR. L. NGENDAKUMANA
- TIME: 3 HOURS

INSTRUCTIONS

Answer both questions in section A and two questions in section B. Total possible mark is 60.

Start **each** question on a new page in your answer booklet.

The marks allocated to **each** question are shown at the end of the section.

Show all your workings.

Credit will be awarded for logical, systematic and neat presentations.

SECTION A

Question 1

(i) The following set of equations describes behavior in the wheat market:

$$Q_{t}^{d} = 120 - 0.5P_{t}$$
$$Q_{t}^{s} = 30 + 0.3P_{t}$$
$$P_{t} = P_{t-1} - \alpha(Q_{t-1}^{s} - Q_{t-1}^{d})$$

Where Q^{d} is quantity demanded, Q^{s} is quantity supplied, P is price and α is a positive parameter.

- a. Solve for the long run equilibrium price and quantity [3]
- b. Solve the first order difference equation in the price and find the particular solution if $P_0 = 200$ and $\alpha = 1$ [7]

(ii) Let Y_t , C_t and I_t denote the national product, consumption and investment respectively, in Zimbabwe at time t. Then at any time:

$$Y_t = C_t + I_t$$

Suppose furthermore that

$$C_t = 1000 + 0.7Y_{t-1}$$

and that $I_t = 500$ for all t.

(i) Deduce the difference equation for Y_t , and solve it for $Y_0 = 2000$ and t=2.

[5]

[2]

Question 2

An individual has m dollars to spend on three commodities X, Y, and Z. The utility enjoyed by consuming X, Y and Z units is given by

 $v(x, y, z) = x^2 y^3 z$ so he is faced with the problem:

Max v(x, y, z) subject to x + y + z = m

- a. Write down the Lagrange function
- b. Find the optimal values of x(m), y(m), and z(m) that solve the maximization problem above and the associated Lagrange multiplier $\lambda(m)$. [7]

c. Let
$$f^*(m) = x(m)^2 y(m)^3 z(m)$$
 be the value function. Verify that $\frac{df^*(m)}{dm} = \lambda(m)$
[3]

d. Using the concept of optimal value function, highlight the economic interpretation of the Lagrange multiplier [3]

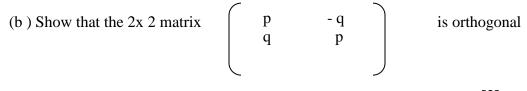
SECTION B

Question 3

(i) (a) Given the matrix below:

A=
$$\begin{pmatrix} a & a^2 - 1 & -3 \\ a + 1 & 2 & a^2 + 4 \\ -3 & 4a & -1 \end{pmatrix}$$

For what values of a is the matrix A symmetric? Compute the matrix A by giving its appropriate value (s). [3]



[2]

(ii) Consider the following linked macroeconomic model of Uganda and Zambia that trade with each other.

$$Y_U = C_U + A_U + X_U - M_U$$

(i)
$$C_U = c_U Y_U$$
$$M_U = m_U Y_U$$

$$Yz = C_z + A_z + X_z - M_z$$

(ii)
$$C_z = c_z Y_z$$
$$M_z = m_z Y_z$$

Where: Y_z represents national income in Zambia

 Y_U represents national income in Uganda

A_z represents (exogenous) autonomous expenditure Zambia

 A_{U} represents (exogenous) autonomous expenditure Uganda

- X_{z} represents exports in Zambia
- X_{U} represents exports in Uganda
- M_{z} represents imports in Zambia
- M_{U} represents imports in Uganda
- C_z represents consumption in Zambia
- C_{II} represents consumption in Uganda
- (a) Interpret the two equations $X_z = M_U$ and $X_U = M_z$ [2]
- (b) Given the equations in part (a), calculate the corresponding equilibrium values of Y_z and Y_U as functions of the exogenous variables. [7]
- (c) How does an increase in A_z affect Y_U ? Interpret your answer. [1]

Question 4

(i) An economy has three industries- fishing, forestry and boat building. To produce one tone of fish requires the services of α fishing boats. To produce one ton of timber requires β tones of fish in order to feed the foresters. To produce one ton of fishing boat requires γ tons of timber. Suppose the final demands for the three goods are 85, 95, and 20 units, respectively. If x_1 , x_2 and x_3 denote the number of units that have to be produced in the three sectors,

- (a) Write down the Leontief model for the problem. [2]
- (b) Find the number of units that has to be produced in each sector in order to meet the final demands. [6]
- (c) What assumption would make sense for this economy to achieve efficient level of production of the three commodities? [2]
- (ii) Use Cramer's Rule to find Y and C when Y = C+IO+GO C = a + bY

Where Y is the national product and C is a private consumption. The symbols Io (private investment), Go (public consumption and investment), a and b all represent constants, with b < 1. (Actually, this is a typical case in which one should not use Cramer's rule, because Y and C can be found much more simply. How? [5]

Question 5

(i) Let Y denote Angola disposable income, C denote consumption, I investment and T denote tax. Suppose that the Angolan economy model for the years 1970-1980 is described by the following equations:

(1) X=93.53 (2) C=0.712Y+95.05 (3) T=0.158 (C+X)-34.30 (4) Y=C+X-T Solve for C, X, Y and T using two different methods of your choice [10]

(ii) After defining both types of vectors, explain how dots vectors are computed. Use concrete examples to support your arguments [5]

.....End of paper....