AFRICA UNIVERSITY

(A United Methodist-Related Institution)

"Investing in Africa's Future"

FACULTY OF MANAGEMENT AND ADMINISTRATION

COURSE TITLE: **MMS 105 MATHEMATICS FOR BUSINESS II (P-Harare)**

SEMESTER II: **FINAL EXAMINATION PARALLEL – NOVEMBER 2013**

LECTURER: **MR T. MAKAMBWA**

TIME: **3 HOURS**

INSTRUCTIONS

Answer **any four questions**

Total possible mark is 100

Start **each** question on a new page in your answer Booklet.

The marks allocated to **each** question are shown at the end of the section.

Question One

a) Solve using matrices the equations $2x-y = 3$, $5x+y = 4$. [5]

b) Solve the equations $2x + 8y + 5z = 5$, $x + y + z = -2$, $x + 2y - z = 2$ by using matrix method. [10]

c) The cost of 2kg of wheat and 1kg of sugar is Rs 7. The cost of 1kg wheat and 1kg of rice is Rs. 7. The cost of 3kg of wheat, 2kg of sugar and 1kg of rice is Rs. 17. Find the cost of each per kg., using matrix method. [10]

Question Two

a) A firm produces x tonnes of output at a total cost

$$C(x) = \frac{x^3}{10} - 4x^2 + 20x + 5$$

Find

(i) Average cost [2]

(ii) Average Variable Cost [2]

(iii) Average Fixed Cost [2]

(iv) Marginal Cost and [3]

(v) Marginal Average Cost. [3]

b) The demand curve for a monopolist is given by $x = 100-4p$

(i) Find the total revenue, average revenue and marginal revenue.

(ii) At what value of x, the marginal revenue is equal to zero? [8]

c) Find the equilibrium price and equilibrium quantity for the following demand and supply functions, $Q_d = 4-0.06p$ and $Q_s = 0.6+0.11p$ [5]

Question Three

a) Investigate the maxima and minima of the function $y= 2x^3 + 3x^2 - 36x + 10$. [8]

b) Find the points of inflection of the curve $y = 2x^4 - 4x^3 + 3$. [6]

c) The total cost and total revenue of a firm are given by $C = x^3 - 12x^2 + 48x + 11$ and $R = 83x - 4x^2 - 21$.

Find the Output

(i) When the revenue is maximum [5]

(ii) When profit is maximum. [6]

Question Four

a) If the demand function is $p = 35 - 2x - x^2$ and the demand x_0 is 3, find the consumers' surplus. [8]

b) If the supply law is $p = 4 - x + x^2$, find the producers' surplus when the price is 6. [8]

c) The demand and supply law under a pure competition are given by $p_d = 23 - x^2$ and $p_s = 2x^2 - 4$. Find the consumers' surplus and producers' surplus at the market equilibrium price. [9]

Question Five

a) On a chicken farm, the poultry is given a healthy diet to gain weight. The chickens have to consume a minimum of 15 units of Substance A and another 15 units of Substance B. In the market there are only two classes of compounds: Type X, with a composition of one unit of A to five units of B, and another type, Y, with a composition of five units of A to one of B. The price of Type X is $10 and Type Y, $30. What are the quantities of each type of compound that have to be purchased to cover the needs of the diet with a minimal cost? [15]

b) There is only 600 milligrams of a certain drug that is needed to make both large and small pills for small scale pharmaceutical distribution. The large tablets weigh 40 milligrams and the small ones, 30 milligrams. Consumer research determines that at least twice the amount of the smaller tablets are needed than the large ones and there needs to be at least three large tablets made. Each large tablet is sold for a profit of $2 and the small tablet, $1. How many tablets of each type have to be prepared to obtain the maximum profit? [10]

Question Six

a) Find the general and particular solutions of the following

i. $P_{t+1} = 0.9P_t + 80$ given $P_1 = 170$

ii. $\mathbf{Y}_{t+1} = 10Y_t + 900$, given $Y_0 = 20$

iii. $\mathbf{P}_{t+1} = 0.75P_t + 9(3)^t$, given $P_0 = 190$

[5+5+5]

b) For each of the differential equations below (i) derive the definite solution, and (ii) use this solution to predict the value of y when $t = 10$.

(a) $\frac{dy}{dt} = 0.2y$ with initial value $y_0 = 200$

(b) $\frac{dy}{dt} = 1.2y$ with initial value $y_0 = 45$

[10]

Question Seven

a) Solve and find a general solution of the equation $y^1e^{-x}+e^{2x}=0$
b) Solve the differential equation $y^1= 12e^{0.6t}$, given y=80 when t=0

c) find the particular solution of $y^1+ 2y =6$,given y=1 when x=0

d) The differential equation dP/dt =0.01P models the relationship between the numbers in a population at t. What is the proportional rate of population growth?

i. Deduce the equation which describes the total population at any time t.
ii. Calculate the numbers in the population in 2200 given that P=58.6 million in 1998(let t=0 at the start of 1998).
iii. Calculate the number of years taken for the population

[5+5+5+10]

Question Eight

i) Differentiate

(a) $Y = (2x + 1)^{10}$

(b) $Y = (x^2+ 3x - 5)^3$

(c) $Y = x^2(x + 5)^3$

(d) $Y = x^5(4x + 5)^2$

(e) $Y = \frac{x^2}{x^2 +4}$

[12.5]

ii) Integrate the following:

a) $\int 30x^4\, dx$
b) $\int(24+ 7.2x^{-2})\, dx$
c) $\int 0.5/x\, dx$
d) $\int(48x-0.4e^{-1.4x})dx$
e) $\int(65+3^x)\, dx$

[12.5]

END OF PAPER

Mathematical Formulae

Integration Techniques

Many integration formulas can be derived directly from their corresponding derivative formulas, while other integration problems require more work. Some that require more work are substitution and change of variables, integration by parts, trigonometric integrals, and trigonometric substitutions.

Basic formulas

Most of the following basic formulas directly follow the differentiation rules.

1. $\int kf(x)\,dx = k\int f(x)\,dx$
2. $\int [f(x) \pm g(x)]\,dx = \int f(x)\,dx \pm \int g(x)\,dx$
3. $\int k\,dx = kx + C$
4. $\int x^n\,dx = \frac{x^{n+1}}{n+1} + C, n \neq -1$
5. $\int \sin x\,dx = -\cos x + C$
6. $\int \cos x\,dx = \sin x + C$
7. $\int \sec^2 x\,dx = \tan x + C$
8. $\int \csc^2 x\,dx = -\cot x + C$
9. $\int \sec x \tan x\,dx = \sec x + C$
10. $\int \csc x \cot x\,dx = -\csc x + C$
11. $\int e^x\,dx = e^x + C$
12. $\int a^x\,dx = \frac{a^x}{\ln a} + C, a > 0, a \neq 1$
13. $\int \frac{dx}{x} = \ln|x| + C$
14. $\int \tan x\,dx = -\ln|\cos x| + C$

15. $\int \cot x\, dx = \ln|\sin x| + C$

16. $\int \sec x\, dx = \ln|\sec x + \tan x| + C$

17. $\int \csc x\, dx = -\ln|\csc x + \cot x| + C$

18. $\int \frac{dx}{\sqrt{a^2 - x^2}} = \arcsin \frac{x}{a} + C$

19. $\int \frac{dx}{a^2 + x^2} = \frac{1}{a} \arctan \frac{x}{a} + C$

20. $\int \frac{dx}{x\sqrt{x^2 - a^2}} = \frac{1}{a} \operatorname{arcsec} \frac{x}{a} + C$

Applications of Integration

∴ Producer Surplus = $p_0q_0 - \int gx\,dx$ with limits from 0 to q_0

Consumer Surplus = $\int gx\,dx - p_0q_0$ with limits from 0 to q_0

Total Cost = ∫ MCdx

Total Revenue =∫MRdx

Differentiation

The chain rule

If *y* is a function of *u*, which is itself a function of *x*, then

$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$

differentiate the outer function and multiply by the derivative of the inner function

The product rule

If *y* = *uv* then = $\cdot \frac{dy}{dx} = u.\frac{dv}{dx} \times v.\frac{du}{dx}$

This rule tells you how to differentiate the product of two functions: **multiply each function by the derivative of the other and add**

The quotient rule

If *y* = $\frac{u}{v}$ then $\frac{dy}{dx} = \frac{v.du/dx - u.dv/dx}{v^2}$

This rule tells you how to differentiate the quotient of two functions: **bottom times derivative of top, minus top times derivative of bottom, all over botto**