AFRICA UNIVERSITY

(A United Methodist-Related Institution)

"Investing in Africa's Future"

FACULTY OF MANAGEMENT AND ADMINISTRATION

COURSE TITLE: MMS101 : MATHEMATICS FOR BUSINESS I

SEMESTER 2: FINAL EXAMINATION NOV-DEC 2013

LECTURER: MR A. KANDIERO

TIME: 3 HOURS

INSTRUCTIONS

Answer **questions as instructed in each section.**
Total possible mark is **100.**

Start **each section B** question on a new page in your answer booklet.

The marks allocated to **each** question are shown at the end of the section.

Show all your workings.

Credit will be awarded for logical, systematic and neat presentations.

SECTION A – **ANSWER ALL QUESTIONS [40 marks]**

A1. [5]

$$\frac{\left(\frac{3}{x}\right)}{x+3} = \frac{3}{x(x+3)}$$

A2. [5]

$$\frac{\left(\frac{5Q}{P+2}\right)}{\left(\frac{1}{(P+2)}\right)}$$

A3. [10]

Solve the equations

(a) $20x - 3x^2 = 10(2x - 3)$ (b) $\frac{2}{x} = \frac{x}{2x} + 1$

45. [15]

Graph the lines given by the equations

(a) $y = 44 - 5x$ (ii) $y = 2x + 2$ **[5]**

(b) Solve the simultaneous equations (i) and (ii).
What does the solution mean? **[5]**

(c) Indicate the solution on the graph in (a).**[5]**

A6. [5]

The demand and supply functions for a good are given by the equations

$P = 80 - 2Q$ and $P = 20 + 4Q$ respectively.

(a) Calculate the equilibrium price and quantity

PART 2 - ANSWER ANY **THREE** QUESTIONS **[60 marks]**

Question 1: Linear budget constraint

Pocket money, £5, may be spent on either ice-cream or soft drinks. Ice-cream costs 12p per unit while drinks cost 20p per unit.

(a) Write down the equation of the budget constraint. Graph the constraint.

(b) Show by calculation and graphically how the budget constraint changes if the price of ice-cream drops to 9p, while pocket money and the price of drinks do not change.

(c) Show by calculation and graphically how the budget constraint changes if the price of soft drinks increase to 25p, while pocket money and the price of ice-cream do not change.

(d) If pocket money increases to £7.50, and the price of ice-cream and drinks remain the same, how does the budget constraint alter? Graph the new budget constraint.

Question 2 : Financial Mathematics (20 Marks)

Interest compounded at various intervals. APR

1. If £1000 is invested at a 9% nominal rate of interest, determine,
(a) the value of the investment after 20 years when interest is compounded continuously. **[6]**
(b) How many years will it take for the value of the investment to reach £4000, when continuous compounding is used? **[6]**
(c) What is the present value of £1000 which will be paid five years from now, if interest of 9% is compounded continuously? **[8]**

Question 3 : EQUILIBRIUM AND BREAK EVEN POINT (20 Marks)

1. The following demand and supply functions for a safari holiday package are:

$$\text{Demand function:} \quad Q = 81 - 0.05P$$

$$\text{Supply function:} \quad Q = -24 + 0.025P$$

(a) Calculate the equilibrium price and quantity, algebraically
(b) Graph the supply and demand function, showing the equilibrium.

2. (See question 1)
The government imposes a tax of £120.
(a) Write down the equation of the supply function adjusted for tax, hence graph it on the diagram in 1(b).

(b) Calculate the equilibrium price and quantity when the tax is imposed.
(c) Outline the distribution of the tax, i.e. calculate the tax paid by the consumer and by the travel agent.

Question 4 : EQUILIBRIUM AND BREAK EVEN POINT (20 Marks)

1. Solve the following equations:
   (a) $x^2 - 25 = 0$ (b) $x^2 + 20x = 0$ (c) $x^2 - 40x + 14 = 0$
2. A firm charges a fixed price of £80 for each shirt sold. The firm has a total cost function: $TC = Q^3 - 136Q$.
   (a) Write down the equation of the total revenue function.
   (b) Determine the break-even point.

Question 5 : NPV (20 Marks)

The net cash flow for two projects, A (fast food) and B (amusements), is as follows:

Year	0	1	2	3	4	5
Project A	−420 000	−5 000	122 000	130 000	148 000	150 000
Project B	−95 000	−10 000	−120 000	200 000	110 000	−50 000

(a) Use the net present value criterion to decide which project is the most profitable if a discount rate of (i) 6% and (ii) 8% is used.

THE END

Appendix

❒ Financial mathematics

- **Amount due after t years (future value)**—bringing forward a single payment.

  Simple interest: $P_t = P_0(1 + it)$

  Compound interest (annual): $P_t = P_0(1 + i)^t$

  Compound m times annually: $P_t = P_0\left(1 + \frac{i}{m}\right)^{mt}$

  Continuous compounding: $P_t = P_0 e^{it}$

- **Present value**—of a single payment due in t years from now.

  Simple discounting: $P_0 = \frac{P_t}{1 + it} = P_t(1 + it)^{-1}$

  Compound discounting: $P_0 = \frac{P_t}{(1 + i)^t} = P_t(1 + i)^{-t}$

  Continuous discounting: $P_0 = P_t e^{-it}$

- Annual percentage rate

  (a) When the nominal rate is compounded m times per year,

  $$APR = \left(1 + \frac{i}{m}\right)^m - 1$$

  (b) When the nominal rate is compounded continuously,

  $$APR = e^i - 1$$

- **Depreciation**

  Straight-line depreciation
  Reducing-balance depreciation, $A_t = A_0(1 - i)^t$

  Present value: $A_0 = \frac{A_t}{(1 - i)^t}$

- **Net present value and *IRR***

  NPV: present value of a future cash flow, discounted at a given discount rate r.
  IRR: the discount rate for which $NPV = 0$. The *IRR* may be estimated graphically or by the formula: