AFRICA UNIVERSITY

(A United Methodist-Related Institution)

"Investing in Africa's Future"

FACULTY OF MANAGEMENT AND ADMINISTRATION

COURSE TITLE: MMS105 : MATHEMATICS FOR BUSINESS II

SEMESTER 2: FINAL EXAMINATION NOV-DEC 2013

LECTURER: MR A. KANDIERO

TIME: 3 HOURS

INSTRUCTIONS

Answer any **5 questions**, each questions carries a total of 20 marks

Total possible mark is **100.**

Start **each** question on a new page in your answer booklet.

The marks allocated to **each** question are shown at the end of the question.

Some questions require graph papers, request from the invigilators.

1. **Applications of difference equation, salary increase model [20]**
   - **a)** What is a differential equation and give example **[3]**
   - **b)** Define differentiation and give example of business application **[3]**
   - **c)** Define integration and give example of business application **[3]**
   - **d)** The difference equation $Y_{t+1} = 1.2Y_t$ models a salary scale in which income increases by 20% pa.

     If income in Year 1 (i.e. $t = 1$) is £9 000

     find income in Years 2, 3, 4 and 5, sketch and comment on the graph **[5]**
   - **e)** Solve $Y_{t+1} - 1.2Y_t = 0$, given $Y_1 = 9\ 000$

     The trial solution is an equation of the form: $Y_t = A \times a^t$

     So, if $Y_t = A \times a^t$

     then $Y_{t+1} = A \times a^{t+1}$

     Find the genera and particular solution **[6]**

2. Linear Programming [20]

A store wants to liquidate 100 of its shirts and 50pairs of pants from last season. They have decided to put together two offers, A and B. Offer A is a package of one shirt and a pair of pants which will sell for $15. Offer B is a package of three shirts and a pair of pants, which will sell for $25. The store does not want to sell less than 10 packages of Offer A and less than 5 of Offer B. How many packages of each do they have to sell to maximize the money generated from the promotion?

3. Integration and Applications [20]

Solve the differential equations

(a) $\dfrac{dy}{dx} = 0.2y$ (b) $\dfrac{dy}{dx} = 120(1 - 0.2y)$

500 tons of farm effluent is released into a river. The amount of effluent (E) present is given by the differential equation $dE/dt = -0.1E$, where t is in hours

(a) Deduce an expression for E in terms of t.

(b) Calculate the amount of effluent present after 5 hours.

(c) Calculate the time taken for the amount of effluent to reduce to 10.50 tons.

4. Difference equations and applications [20]

Given the Cobb-Douglas production functions

$Q = 60L^{0.5}K^{0.3}$

$Q = 60L^{0.6}K^{0.4}$

$Q = 60L^{0.5}K^{0.3}$

i. Calculate the level of output when L=10 K=15 **[5]**

ii. Calculate the level of output when both inputs double, L=20, K=30 **[5]**

iii. Comment on the returns to scale **[5]**

iv. Find the second derivative of the last function with respect to K and L and comment using the law of diminishing returns. **[5]**

5. Differentiation and Applications [20]

A firm has an average cost function $AC = 10 - 3Q + Q^2$.

(a) Write down the equations for TC, MC.

(b) Determine the values of Q at which (i) MC and (ii) AC are minimised.

(c) Plot the AC and MC curves on the same diagram. Confirm algebraically that the curves intersect at the minimum point on the AC curve.

6. Linear algebra – Matrices and applications [20]

Write the following system of equations as an augmented matrix:

$$\begin{aligned} 3x + 3y + 6z &= 12 \\ x - 3y + 5z &= 5 \\ 2x + 10y - 3z &= 0 \end{aligned}$$

(a) Reduce the augmented matrix to upper triangular form, then solve by back substitution.

(b)

Given the following matrices:

$$A = \begin{pmatrix} 2 & -1 \\ 4 & 3 \end{pmatrix}, \quad B = \begin{pmatrix} 0 & 1 \\ -1 & 2 \end{pmatrix}, \quad C = \begin{pmatrix} 1 \\ 5 \\ 3 \end{pmatrix}, \quad D = \begin{pmatrix} 3 & 1 & 2 \\ 0 & 1 & 1 \end{pmatrix}$$

Show that $AB \neq BA$.

7. Miscellaneous concepts [20]

Assume the non-negativity constraint $(x \geq 0, y \geq 0)$ applies in each of the following problems. In questions 1

(a) Graph the inequality constraints.
(b) Shade in the feasible region.
(c) Calculate the corner points of the feasible region.

1. $$\begin{aligned} 3x + 2y &\geq 15 \\ 6x + 9y &\geq 36 \end{aligned}$$

2. **(a) Integrate the following:**

(i) $\int x^4 + 2x \, dx$ **(ii)** $\int \sqrt{x} + 2 \, dx$ **(iii)** $\int \frac{2}{x} + 2x \, dx$

END OF PAPER

Appendix A : Mathematics for Business Formula Sheet

Difference equations

The trial solution is an equation of the form: $Y_t = A \times a^t$

So, if $Y_t = A \times a^t$

then $Y_{t+1} = A \times a^{t+1}$

$Y_t = CF + PI = Y_{t,c} + Y_{t,p}$

$Y_{t,p} = k \times (b)^t$