AFRICA UNIVERSITY

(A United Methodist-Related Institution)

"Investing in Africa's Future"

FACULTY OF MANAGEMENT AND ADMINISTRATION

COURSE TITLE: **MMS202 - Quantitative Analysis 1**

SEMESTER 1: **Final Examination - Conventional**

DATE: **November 2013**

INSTRUCTOR: **Dr. S. Murairwa**

TIME: **3 Hours**

INSTRUCTIONS

Answer **all** questions in Section A and **any three (3)** questions in Section B

Start **each** question on a new page in your answer booklet.

The marks allocated to **each** question are shown at the end of the section.

Show all your workings.

Credit will be given for logical, systematic and neat presentations.

SECTION A: ANSWER ALL QUESTIONS

1. Let A = {set of all children in Mutare}
   B = {set of children aged 7 years and above}
   C = {set of children aged 6 years and below}
   D = {set of boy children}
   Draw a Venn diagram to display A∩B, B∩C and A∩B∩D **[3 Marks]**

2. Telecom Company frequently receives reports on problem with the telephone line. The following data shows the time (in minutes) used to solve these problems from the customers' lines.

8.9	8.4	7.3	4.0	6.7	4.5	6.3
9.7	3.3	1.1	8.4	4.3	7.6	3.5

   Calculate and interpret:
   i) Coefficient of skewness **[11 Marks]**
   ii) Semi inter-quartile range **[6 Marks]**

3. A sample of Quantitative analysis I students were asked each to select a toy from a box and the colour of each toy selected noted. The resulting sample follows (B = Blue, Gr = Green, R = Red, Ye = Yellow, G = Grey, Bl = Black):

B	B	G	Gr	Gr	G	Gr	B
Gr	B	B	R	Gr	B	B	Gr
R	Gr	R	Bl	B	B	B	R
R	Bl	B	Gr	G	Bl	R	R
Ye	B	B	B	B	Ye	Ye	G
B	B	G	Gr	Gr	G	Gr	B
Gr	B	B	R	Gr	B	B	Gr
R	Gr	R	Bl	B	B	B	R
R	Bl	B	Gr	G	Bl	R	R
Ye	B	B	B	B	Ye	Ye	G

   (a) Construct a frequency distribution table for the data with the relative frequency and percentage relative frequency **[12 Marks]**

   (b) Given the following two data sets:

   A - 12 22 24 22 34 31 26 35 27 39 49 10
   B - 45 36 23 16 37 28 18 13 10 23 30 31

   i) Construct a back-to-back stem and leaf plot **[6 Marks]**
   ii) State two advantages of the stem and leaf plot over a grouped frequency distribution

   [2 Marks]

SECTION B: ANSWER ANY THREE (3) QUESTIONS

4. Attempt the following questions:

   a) In the nursing home study, the researcher found that 12 out of 34 small nursing homes had a resident vaccination rate of less than 80%, while 17 out of 24 large nursing homes had a vaccination rate of less than 80%. Find the 95% confidence interval for the difference of the proportions for the data **[5 Marks]**

   b) One small farm has 10 animals where 6 of them are goats and the rest are sheep. A veterinary wants to study on a disease that attacks the animals. Therefore, she randomly selects without replacement two animals as a sample from the farm. Let X be the random sample representing the number of goats selected in the sample.

   i) Construct and plot a probability distribution of X **[6 Marks]**

   ii) Construct and plot a cumulative probability distribution of X **[3 Marks]**

   iii) Calculate the standard deviation of X **[6 Marks]**

5. Probability is the likelihood of an event occurring.

   a) Bag A contains 4 statistics books and 3 mathematics books. Bag B contains 2 statistics books and 1 mathematics books. Bag C contains 2 statistics books and 3 mathematics books. One book is randomly selected from bag A. If the selected book is statistics, it will be put into bag B and if mathematics book is selected, it will be put into bag C. Then one book will be selected again.

      i) Illustrate the event in a tree diagram **[5 Marks]**

      ii) Find the probability that statistics book was selected **[3 Marks]**

      iii) Probability that mathematics book was selected from bag B or C **[3 Marks]**

   b) Find the weighted mean for a particular student's scores on three exams if the first one was worth 75 points and the student received a score of 70%, the second was worth 50 points and the student received a score of 80%, and the third was worth 30 points and the student received a score of 95%? **[3 Marks]**

   c) In a department store there are 120 customers, 90 of them will buy at least one item. If 4 customers are selected at random, one by one, use complimentary rule to find the probability that at least one of the customers will buy at least one item. Would you consider this event likely to occur? Explain your answer **[6 Marks]**

6. Statistical hypothesis testing is a decision making process for evaluating claims about a population.

   a) The local high school prints baseball cards. The local high school claims that 30%, 60% and 10% of the cards are rookies, veterans and All-Stars respectively. The cards are sold in packages of 100. Suppose a randomly-selected package of cards has 50 rookies, 45 veterans, and 5 All-Stars. Is this consistent with local high school's claim at 5% level of significance? [**6 Marks**]

b) Forty-four sixth graders were divided into 22 matched pairs, each pair having equal IQ's. One member of each pair was randomly selected to receive special training. Then, all of the students were given an IQ test. Test results are summarized below:

Pair	Training	No training
1	95	90
2	89	85
3	76	73
4	92	90
5	91	90
6	53	53
7	67	68
8	88	90
9	75	78
10	85	89
11	90	95

Pair	Training	No training
12	85	83
13	87	83
14	85	83
15	85	82
16	68	65
17	81	79
18	84	83
19	71	60
20	46	47
21	75	77
22	80	83

i) Do these results provide evidence that the special training helped or hurt student performance? Use a 0.05 level of significance [**10 Marks**]
ii) Construct a 95 % confidence interval [**4 Marks**]

7. A company produces goods and services:

(a) The production machine produces defective items at a rate of 0.2. A researcher selects 12 items from the production line. Calculate
i) the probability of more than three defectives in the selected items [**4 Marks**]
ii) the mean and standard deviation of the distribution [**3 Marks**]

(b) The company receives an average of three calls per five-minute period of the working day. In a randomly selected five-minute period of Monday, calculate
i) the probability of receiving four calls [**3 Marks**]
ii) the probability of receiving no call in quarter of an hour of Tuesday [**3 Marks**]
iii) the mean and standard deviation of the probability distribution in (ii) [**2 Marks**]

(c) Let x denote the time it takes to run a road race. Supposed x is approximately normally distributed with mean of 190 minutes and variance of 441 minutes. If one runner is selected at random, what is the probability that the runner will complete the road race:
a) in less than 150 minutes? [**2 Marks**]
b) in 205 to 245 minutes? [**3 Marks**]

The end of paper

ADDITIONAL INFORMATION

1. Sturge's Rule:

$$C = 1 + 3.3\log(n)$$

Class width, $i > \frac{range}{C}$

2. Mean of grouped data $= \frac{\sum_{i=1}^{n} fx_i}{n}$

3. Mean of ungrouped data $= \frac{\sum_{i=1}^{n} x_i}{n}$

4. Mode = $L_{mo} + \left(\frac{\Delta_1}{\Delta_1 + \Delta_2}\right) i$

5 Median = $L_{me} + \left(\frac{\frac{n}{2} - F}{f_m}\right) i$

6. Standard deviation $= \sqrt{\frac{\sum_{i=1}^{n} fx_i^2 - \frac{\left(\sum_{n=1}^{n} fx_i\right)^2}{n}}{n-1}}$

7. Standard Deviation of ungrouped data= $\sqrt{\frac{\sum_{i=1}^{n} x_i^2 - \frac{\left(\sum_{i=1}^{n} x_i\right)^2}{n}}{n-1}}$

8. Coefficient of skewness: $S_k = \frac{3(mean - median)}{s} = \frac{mean - \mathrm{mod}\, e}{s}$

9. Conditional probability: $P(A¿) = \frac{P(A \cap B)}{P(A)}$

10. Binomial Distribution: $P(X = x) = n C_x p^x q^{n-x}$

11. Poisson Distribution: $P(X = x) = \frac{e^{-\lambda} \lambda^x}{x!}$

12. Test statistic: $t = \frac{\acute{X} - \mu}{\frac{s}{\sqrt{n}}}$ or $Z = \frac{\acute{X} - \mu}{\frac{\sigma}{\sqrt{n}}}$

13. Proportion Test statistic = $Z = \frac{\hat{p} - p}{\sqrt{pq/n}}$

14. $Z = \frac{X - \mu}{\sigma}$

15. Weighted Mean: $\acute{X}_w = \frac{\sum xw}{\sum w}$

16. CV = $\frac{s}{\acute{x}} \times 100$

17. $\hat{p} - z_{\alpha/2}\sqrt{\frac{\hat{p}\hat{q}}{n}} < p < \hat{p} + z_{\alpha/2}\sqrt{\frac{\hat{p}\hat{q}}{n}}$

18. $E(X) = \sum X . P(x)$

19. $\sigma = \sqrt{\sum x^2 P(x) - \mu^2}$

20. $\bar{X} - z_{\alpha/2}\left(\frac{\sigma}{\sqrt{n}}\right) < \mu < \bar{X} + z_{\alpha/2}\left(\frac{\sigma}{\sqrt{n}}\right)$

21. $z = \frac{(\bar{X}_1 - \bar{X}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$ or $t = \frac{(\bar{X}_1 - \bar{X}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$

22. $(\bar{X}_1 - \bar{X}_2) - z_{\alpha/2}\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}} < (\mu_1 - \mu_2)$

$$< (\bar{X}_1 - \bar{X}_2) + z_{\alpha/2}\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$$

23. $t = \frac{\bar{D} - \mu_D}{s_D / \sqrt{n}}$

24. $z = \frac{(\hat{p}_1 - \hat{p}_2) - (p_1 - p_2)}{\sqrt{\bar{p}\bar{q}\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$

where

$\bar{p} = \frac{X_1 + X_2}{n_1 + n_2}$ $\quad \hat{p}_1 = \frac{X_1}{n_1}$

$\bar{q} = 1 - \bar{p}$ $\quad \hat{p}_2 = \frac{X_2}{n_2}$

25. $\chi^2 = \sum \frac{(O - E)^2}{E}$

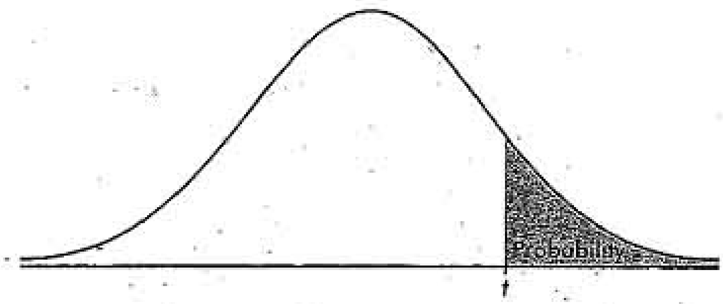


TABLE B: *t*-DISTRIBUTION CRITICAL VALUES

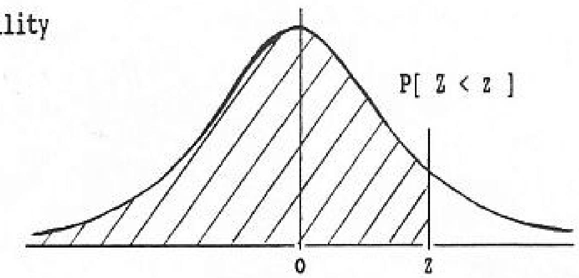
	Tail probability *p*											
df	.25	.20	.15	.10	.05	.025	.02	.01	.005	.0025	.001	.0005
1	1.000	1.376	1.963	3.078	6.314	12.71	15.89	31.82	63.66	127.3	318.3	636.6
2	.816	1.061	1.386	1.886	2.920	4.303	4.849	6.965	9.925	14.09	22.33	31.60
3	.765	.978	1.250	1.638	2.353	3.182	3.482	4.541	5.841	7.453	10.21	12.92
4	.741	.941	1.190	1.533	2.132	2.776	2.999	3.747	4.604	5.598	7.173	8.610
5	.727	.920	1.156	1.476	2.015	2.571	2.757	3.365	4.032	4.773	5.893	6.869
6	.718	.906	1.134	1.440	1.943	2.447	2.612	3.143	3.707	4.317	5.208	5.959
7	.711	.896	1.119	1.415	1.895	2.365	2.517	2.998	3.499	4.029	4.785	5.408
8	.706	.889	1.108	1.397	1.860	2.306	2.449	2.896	3.355	3.833	4.501	5.041
9	.703	.883	1.100	1.383	1.833	2.262	2.398	2.821	3.250	3.690	4.297	4.781
10	.700	.879	1.093	1.372	1.812	2.228	2.359	2.764	3.169	3.581	4.144	4.587
11	.697	.876	1.088	1.363	1.796	2.201	2.328	2.718	3.106	3.497	4.025	4.437
12	.695	.873	1.083	1.356	1.782	2.179	2.303	2.681	3.055	3.428	3.930	4.318
13	.694	.870	1.079	1.350	1.771	2.160	2.282	2.650	3.012	3.372	3.852	4.221
14	.692	.868	1.076	1.345	1.761	2.145	2.264	2.624	2.977	3.326	3.787	4.140
15	.691	.866	1.074	1.341	1.753	2.131	2.249	2.602	2.947	3.286	3.733	4.073
16	.690	.865	1.071	1.337	1.746	2.120	2.235	2.583	2.921	3.252	3.686	4.015
17	.689	.863	1.069	1.333	1.740	2.110	2.224	2.567	2.898	3.222	3.646	3.965
18	.688	.862	1.067	1.330	1.734	2.101	2.214	2.552	2.878	3.197	3.611	3.922
19	.688	.861	1.066	1.328	1.729	2.093	2.205	2.539	2.861	3.174	3.579	3.883
20	.687	.860	1.064	1.325	1.725	2.086	2.197	2.528	2.845	3.153	3.552	3.850
21	.686	.859	1.063	1.323	1.721	2.080	2.189	2.518	2.831	3.135	3.527	3.819
22	.686	.858	1.061	1.321	1.717	2.074	2.183	2.508	2.819	3.119	3.505	3.792
23	.685	.858	1.060	1.319	1.714	2.069	2.177	2.500	2.807	3.104	3.485	3.768
24	.685	.857	1.059	1.318	1.711	2.064	2.172	2.492	2.797	3.091	3.467	3.745
25	.684	.856	1.058	1.316	1.708	2.060	2.167	2.485	2.787	3.078	3.450	3.725
26	.684	.856	1.058	1.315	1.706	2.056	2.162	2.479	2.779	3.067	3.435	3.707
27	.684	.855	1.057	1.314	1.703	2.052	2.158	2.473	2.771	3.057	3.421	3.690
28	.683	.855	1.056	1.313	1.701	2.048	2.154	2.467	2.763	3.047	3.408	3.674
29	.683	.854	1.055	1.311	1.699	2.045	2.150	2.462	2.756	3.038	3.396	3.659
30	.683	.854	1.055	1.310	1.697	2.042	2.147	2.457	2.750	3.030	3.385	3.646
40	.681	.851	1.050	1.303	1.684	2.021	2.123	2.423	2.704	2.971	3.307	3.551
50	.679	.849	1.047	1.299	1.676	2.009	2.109	2.403	2.678	2.937	3.261	3.496
60	.679	.848	1.045	1.296	1.671	2.000	2.099	2.390	2.660	2.915	3.232	3.460
80	.678	.846	1.043	1.292	1.664	1.990	2.088	2.374	2.639	2.887	3.195	3.416
100	.677	.845	1.042	1.290	1.660	1.984	2.081	2.364	2.626	2.871	3.174	3.390
1000	.675	.842	1.037	1.282	1.646	1.962	2.056	2.330	2.581	2.813	3.098	3.300
∞	.674	.841	1.036	1.282	1.645	1.960	2.054	2.326	2.576	2.807	3.091	3.291
	50%	60%	70%	80%	90%	95%	96%	98%	99%	99.5%	99.8%	99.9%
	Confidence level *C*											

STANDARD STATISTICAL TABLES

1. Areas under the Normal Distribution

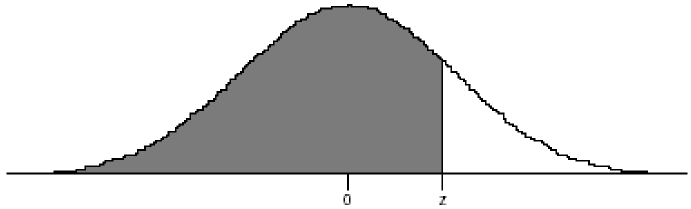
The table gives the cumulative probability up to the standardised normal value z i.e.

$$P[\ Z < z\ ] = \int_{-\infty}^{z} \frac{1}{\sqrt{2\pi}} \exp(-\tfrac{1}{2}Z^2)\ dZ$$



z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5159	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7854
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8804	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9773	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9865	0.9868	0.9871	0.9874	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9924	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9980	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986

z	3.00	3.10	3.20	3.30	3.40	3.50	3.60	3.70	3.80	3.90
P	0.9986	0.9990	0.9993	0.9995	0.9997	0.9998	0.9998	0.9999	0.9999	1.0000



Normal Deviate z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
-4.0	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
-3.9	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
-3.8	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
-3.7	.0001	.0001	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
-3.6	.0002	.0002	.0001	.0001	.0001	.0001	.0001	.0001	.0001	.0001
-3.5	.0002	.0002	.0002	.0002	.0002	.0002	.0002	.0002	.0002	.0002
-3.4	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0002
-3.3	.0005	.0005	.0005	.0004	.0004	.0004	.0004	.0004	.0004	.0003
-3.2	.0007	.0007	.0006	.0006	.0006	.0006	.0006	.0005	.0005	.0005
-3.1	.0010	.0009	.0009	.0009	.0008	.0008	.0008	.0008	.0007	.0007
-3.0	.0013	.0013	.0013	.0012	.0012	.0011	.0011	.0011	.0010	.0010
-2.9	.0019	.0018	.0018	.0017	.0016	.0016	.0015	.0015	.0014	.0014
-2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
-2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
-2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
-2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0048
-2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
-2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
-2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
-2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
-2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
-1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
-1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
-1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
-1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
-1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
-1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0721	.0708	.0694	.0681
-1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
-1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
-1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
-1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
-.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
-.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
-.7	.2420	.2389	.2358	.2327	.2296	.2266	.2236	.2206	.2177	.2148
-.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
-.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
-.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
-.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483

Chi Squared Distribution Table

df	0.995	0.99	0.975	0.95	0.9	0.1	0.05	0.025	0.01	0.005
1	0	0	0.001	0.004	0.016	2.706	3.841	5.024	6.635	7.879
2	0.01	0.02	0.051	0.103	0.211	4.605	5.991	7.378	9.21	10.597
3	0.072	0.115	0.216	0.352	0.584	6.251	7.815	9.348	11.345	12.838
4	0.207	0.297	0.484	0.711	1.064	7.779	9.488	11.143	13.277	14.86
5	0.412	0.554	0.831	1.145	1.61	9.236	11.07	12.833	15.086	16.75
6	0.676	0.872	1.237	1.635	2.204	10.645	12.592	14.449	16.812	18.548
7	0.989	1.239	1.69	2.167	2.833	12.017	14.067	16.013	18.475	20.278
8	1.344	1.646	2.18	2.733	3.49	13.362	15.507	17.535	20.09	21.955
9	1.735	2.088	2.7	3.325	4.168	14.684	16.919	19.023	21.666	23.589
10	2.156	2.558	3.247	3.94	4.865	15.989	18.307	20.483	23.209	25.188
11	2.603	3.053	3.816	4.575	5.578	17.275	19.675	21.92	24.725	26.757
12	3.074	3.571	4.404	5.226	6.304	18.549	21.026	23.337	26.217	28.3
13	3.565	4.107	5.009	5.892	7.042	19.812	22.362	24.736	27.688	29.819
14	4.075	4.66	5.629	6.571	7.79	21.064	23.685	26.119	29.141	31.319
15	4.601	5.229	6.262	7.261	8.547	22.307	24.996	27.488	30.578	32.801
16	5.142	5.812	6.908	7.962	9.312	23.542	26.296	28.845	32	34.267
17	5.697	6.408	7.564	8.672	10.085	24.769	27.587	30.191	33.409	35.718
18	6.265	7.015	8.231	9.39	10.865	25.989	28.869	31.526	34.805	37.156
19	6.844	7.633	8.907	10.117	11.651	27.204	30.144	32.852	36.191	38.582
20	7.434	8.26	9.591	10.851	12.443	28.412	31.41	34.17	37.566	39.997
21	8.034	8.897	10.283	11.591	13.24	29.615	32.671	35.479	38.932	41.401
22	8.643	9.542	10.982	12.338	14.041	30.813	33.924	36.781	40.289	42.796
23	9.26	10.196	11.689	13.091	14.848	32.007	35.172	38.076	41.638	44.181
24	9.886	10.856	12.401	13.848	15.659	33.196	36.415	39.364	42.98	45.559
25	10.52	11.524	13.12	14.611	16.473	34.382	37.652	40.646	44.314	46.928
26	11.16	12.198	13.844	15.379	17.292	35.563	38.885	41.923	45.642	48.29
27	11.808	12.879	14.573	16.151	18.114	36.741	40.113	43.195	46.963	49.645
28	12.461	13.565	15.308	16.928	18.939	37.916	41.337	44.461	48.278	50.993
29	13.121	14.256	16.047	17.708	19.768	39.087	42.557	45.722	49.588	52.336
30	13.787	14.953	16.791	18.493	20.599	40.256	43.773	46.979	50.892	53.672
40	20.707	22.164	24.433	26.509	29.051	51.805	55.758	59.342	63.691	66.766
50	27.991	29.707	32.357	34.764	37.689	63.169	67.505	71.42	76.154	79.49
60	35.534	37.485	40.482	43.188	46.459	74.397	79.082	83.298	88.379	91.952
70	43.275	45.442	48.758	51.739	55.329	85.527	90.531	95.023	100.425	104.215
80	51.192	53.54	57.153	60.391	64.278	96.578	101.879	106.629	112.329	116.321
90	59.196	61.754	65.647	69.126	73.291	107.565	113.145	118.136	124.116	128.294
100	67.328	70.065	74.222	77.929	82.358	118.498	124.342	129.561	135.807	140.169