AFRICA UNIVERSITY

(A United Methodist-Related Institution)

"Investing in Africa's Future"

FACULTY OF MANAGEMENT AND ADMINISTRATION

COURSE TITLE: MMS202 Quantitative Analysis 1

SEMESTER 1: Final Examination November 2016 (Conventional)

LECTURER: Dr. S. Murairwa

TIME: 3 Hours

INSTRUCTIONS

Answer All questions.

Start each question on a new page in your answer booklet.

The marks allocated are shown at the end of each question.

Show all your workings.

Credit will be given for logical, systematic and neat presentations.

1. The weekly wages of company employees in US dollars ($) are given in the following table:

No.	Wage	Gender	No.	Wage	Gender
1	236	F	26	334	F
2	573	M	27	600	F
3	660	F	28	592	M
4	1005	M	29	728	M
5	513	M	30	125	F
6	188	F	31	401	F
7	252	F	32	759	F
8	200	F	33	1342	M
9	469	F	34	324	F
10	191	F	35	337	F
11	675	M	36	1406	M
12	392	F	37	530	M
13	346	F	38	644	M
14	264	F	39	776	F
15	363	F	40	440	F
16	344	F	41	548	F
17	949	M	42	751	F
18	490	M	43	618	F
19	745	F	44	822	M
20	2033	M	45	437	F
21	391	F	46	293	F
22	179	F	47	995	M
23	1629	M	48	446	F
24	552	F	49	1432	M
25	144	F	50	901	F

a) Calculate the interquartile range **[7 Marks]**
b) Construct a gender frequency distribution of the wages **[6 Marks]**
c) Construct a gender relative frequency distribution of the weekly wages of the company employees **[3 Marks]**
d) Use the total frequency in (b) to calculate
i) Mean **[2 Marks]**
ii) Median **[4 Marks]**
iii) Mode **[3 Marks]**
iv) Standard Deviation **[4 Marks]**
v) Range **[2 Marks]**
vi) Use the relationship among the measures of central location to determine the distribution of the employees' weekly wages. Verify your answer **[6 Marks]**

2. The ages of 5 randomly selected members of a club are:

42 52 57 63 51

Calculate the sample

(a) Mean [2 Marks]
(b) Median [2 Marks]
(c) Standard Deviation [3 Marks]
(d) Range [2 Marks]
(e) What is the distribution of the ages? [3 Marks]

3. Attempt the following questions:

a) Suppose that the number of calls to the 911 emergency number between 8:00 and 8:30 PM on Fridays is a Poisson random variable X with $\lambda = 3.5$.
   i) Calculate the probability of receiving 3 calls [2 Marks]
   ii) Show that over 97 percent of the probability mass is already accounted for by $x \leq 7$ even though x ranges to infinity [6 Marks]
   iii) Find the mean and standard deviation of the probability distribution [2 Marks]

b) You go to see the doctor about an ingrowing toenail. The doctor selects you at random to have a blood test for swine flu, which for the purposes of this exercise we will say is currently suspected to affect 1 in 10 000 people in country U. The test is 99% accurate, in the sense that the probability of a false positive is 1%. The probability of a false negative is zero. You test positive.
   i) What is the new probability that you have swine flu? [5 Marks]
   ii) Now imagine that you went to a friend's wedding in country V recently and it is known that 1 in 200 people who visited country V recently come back with swine flu. Given the same test result as above, what should your revised estimate be for the probability you have the disease? [3 Marks]

c) The distribution of employees is presented in a table below

Category	Graduate (G)	Not graduate ($\bar{G}$)
Male (M)	7	20
Female (F)	4	9

If one of the employees is selected at random, find the probability that

   i) the employee is a female and a graduate [2 Marks]
   ii) the employee is not a graduate [2 Marks]
   iii) the employee is not a graduate given that she is female [3 Marks]
   iv) the employee is a male given that he is a graduate [3 Marks]

4. Suppose that the process by which the shoes are manufactured generates the following population probability distribution for the three values that the random variable X can take:

X	0	1	2	3	4
$P(x)$	0.015	0.235	0.425	0.245	0.080

a) Plot the data to determine the distribution **[3 Marks]**
b) Calculate the mean and standard deviation of the probability distribution **[5 Marks]**
c) Construct a cumulative probability distribution **[2 Marks]**

5. An education officer claims that the average salary of temporary teachers in district X is less than $60 per day. A random sample of eight schools is selected and the daily salaries are shown below.

60 56 60 55 70 55 60 55

(a) Use traditional method to test whether there is enough evidence to support the educator's claim at $\alpha = 0.10$ **[6 Marks]**
(b) Use confidence interval approach to verify your result in (a) **[4 Marks]**
(c) Explain how you would use the p-value method to test the educator's claim in (a) **[3 Marks]**

End of paper

ADDITIONAL INFORMATION

1. Sturge's Rule:

Number of class, $C = 1 + 3.3\log n$

Class width, $i > \dfrac{range}{C}$

2. Mean of grouped data $= \dfrac{\sum_{i=1}^{n} fx_i}{n}$

3. Mean of ungrouped data $= \dfrac{\sum_{i=1}^{n} x_i}{n}$

4. Mode $= L_{mo} + \left(\dfrac{\Delta_1}{\Delta_1 + \Delta_2}\right)i$

5 Median $= L_{me} + \left(\dfrac{\frac{n}{2} - F}{f_m}\right)i$

6. Standard deviation: $s = \sqrt{\dfrac{\sum_{i=1}^{n} fx_i^2 - \dfrac{\left(\sum_{i=1}^{n} fx_i\right)^2}{n}}{n-1}}$

7. Standard Deviation of ungrouped data: $s = \sqrt{\dfrac{\sum_{i=1}^{n} x_i^2 - \dfrac{\left(\sum_{i=1}^{n} x_i\right)^2}{n}}{n-1}}$

8. Coefficient of skewness: $S_k = \dfrac{3(mean - median)}{s} = \dfrac{mean - mode}{s}$

9. Conditional probability: $P(A \setminus B) = \dfrac{P(A \cap B)}{P(A)}$

10. Binomial Distribution

- $P(X = x) = nC_x p^x q^{n-x}$

11. Poisson Distribution

- $P(X = x) = \frac{e^{-\lambda}\lambda^x}{x!}$

12. Hypothesis testing (single mean)

- $Z = \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}}$
- $t = \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}}$, $df = n - 1$

13. Hypothesis testing (single proportion)

- $Z = \frac{p - \pi}{\sqrt{\frac{\pi(1-\pi)}{n}}}$

14. Hypothesis testing (difference of two means)

- $Z = \frac{\bar{X}_1 - \bar{X}_2}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$
- $t = \frac{\bar{X}_1 - \bar{X}_2}{\sqrt{\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}}}$, *df* = smaller $(n_1 - 1;\ n_2 - 1)$
- $t = \frac{\bar{X}_1 - \bar{X}_2}{\sqrt{\frac{S_p^2}{n_1} + \frac{S_p^2}{n_2}}}$

  Where $S_p^2 = \frac{S_1^2(n_1) + S_2^2(n_2)}{n_1 + n_2 - 2}$, *df* = $n_1 + n_2 - 2$
- $t = \frac{\bar{D} - \mu_D}{\frac{S_D}{\sqrt{n}}}$, *df* = n – 1

15. Hypothesis testing (difference of two proportions)
   - $Z = \frac{p_1 - p_2}{\sqrt{pq\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$

16. Confidence Interval (Single mean)
   - $\bar{X} - Z_{\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}} \le \mu \le \bar{X} + Z_{\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}}$
   - $\bar{X} - t_{\frac{\alpha}{2}} \frac{S}{\sqrt{n}} \le \mu \le \bar{X} + t_{\frac{\alpha}{2}} \frac{S}{\sqrt{n}}$

17. Confidence Interval (Difference of two means)
   - $(\bar{X}_1 - \bar{X}_2) - Z_{\frac{\alpha}{2}} \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}} \le (\mu_1 - \mu_2) \le (\bar{X}_1 - \bar{X}_2) + Z_{\frac{\alpha}{2}} \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$
   - $(\bar{X}_1 - \bar{X}_2) - t_{\frac{\alpha}{2}} \sqrt{\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}} \le (\mu_1 - \mu_2) \le (\bar{X}_1 - \bar{X}_2) + t_{\frac{\alpha}{2}} \sqrt{\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}}$

     df = smaller $(n_1 - 1;\ n_2 - 1)$
   - $(\bar{X}_1 - \bar{X}_2) - t_{\frac{\alpha}{2}} \sqrt{\frac{S_p^2}{n_1} + \frac{S_p^2}{n_2}} \le (\mu_1 - \mu_2) \le (\bar{X}_1 - \bar{X}_2) + t_{\frac{\alpha}{2}} \sqrt{\frac{S_p^2}{n_1} + \frac{S_p^2}{n_2}}$

     Where $S_p^2 = \frac{S_1^2(n_1) + S_2^2(n_2)}{n_1 + n_2 - 2}$, *df* = $n_1 + n_2 - 2$

18. Confidence Interval (Single proportion)
   - $p - Z_{\frac{\alpha}{2}} \sqrt{\frac{pq}{n}} \le \pi \le p + Z_{\frac{\alpha}{2}} \sqrt{\frac{pq}{n}}$

19. Confidence Interval (Difference of two proportions)
   - $(p_1 - p_2) - Z_{\frac{\alpha}{2}} \sqrt{\frac{p_1 q_1}{n_1} + \frac{p_2 q_2}{n_2}} \le (\pi_1 - \mu_2) \le (p_1 - p_2) + Z_{\frac{\alpha}{2}} \sqrt{\frac{p_1 q_1}{n_1} + \frac{p_2 q_2}{n_2}}$

20. $Z = \frac{X - \mu}{\sigma}$

21. Weighted Mean: $\bar{X}_w = \frac{\sum xw}{\sum w}$

22. $P(B_i / C) = \dfrac{P(C / B_i)P(B_i)}{\sum_{i=1}^{n} P(C / B_i)P(B_i)}$,

23. $y = \beta_0 + \beta_1 x + e$,

$\beta_1 = \frac{n\sum xy - \sum x \sum y}{n\sum y^2 - (\sum y)^2}$

$\beta_0 = \bar{y} + \beta_1 \bar{x}$

$$r = \frac{n\sum xy - \sum x \sum y}{\sqrt{\{(n\sum x^2 - (\sum x)^2)(n\sum y^2 - (\sum y)^2)\}}}$$

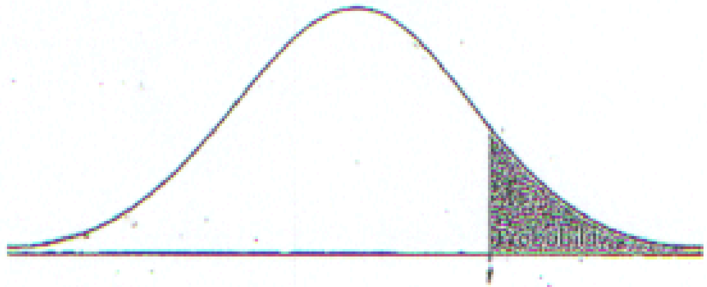


TABLE B: *t*-DISTRIBUTION CRITICAL VALUES

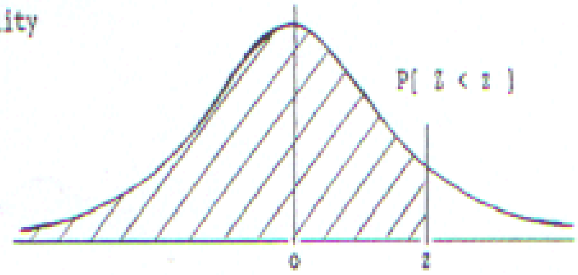
	Tail probability *p*											
df	.25	.20	.15	.10	.05	.025	.02	.01	.005	.0025	.001	.0005
1	1.000	1.376	1.963	3.078	6.314	12.71	15.89	31.82	63.66	127.3	318.3	636.6
2	.816	1.061	1.386	1.886	2.920	4.303	4.849	6.965	9.925	14.09	22.33	31.60
3	.765	.978	1.250	1.638	2.353	3.182	3.482	4.541	5.841	7.453	10.21	12.92
4	.741	.941	1.190	1.533	2.132	2.776	2.999	3.747	4.604	5.598	7.173	8.610
5	.727	.920	1.156	1.476	2.015	2.571	2.757	3.365	4.032	4.773	5.893	6.869
6	.718	.906	1.134	1.440	1.943	2.447	2.612	3.143	3.707	4.317	5.208	5.959
7	.711	.896	1.119	1.415	1.895	2.365	2.517	2.998	3.499	4.029	4.785	5.408
8	.706	.889	1.108	1.397	1.860	2.306	2.449	2.896	3.355	3.833	4.501	5.041
9	.703	.883	1.100	1.383	1.833	2.262	2.398	2.821	3.250	3.690	4.297	4.781
10	.700	.879	1.093	1.372	1.812	2.228	2.359	2.764	3.169	3.581	4.144	4.587
11	.697	.876	1.088	1.363	1.796	2.201	2.328	2.718	3.106	3.497	4.025	4.437
12	.695	.873	1.083	1.356	1.782	2.179	2.303	2.681	3.055	3.428	3.930	4.318
13	.694	.870	1.079	1.350	1.771	2.160	2.282	2.650	3.012	3.372	3.852	4.221
14	.692	.868	1.076	1.345	1.761	2.145	2.264	2.624	2.977	3.326	3.787	4.140
15	.691	.866	1.074	1.341	1.753	2.131	2.249	2.602	2.947	3.286	3.733	4.073
16	.690	.865	1.071	1.337	1.746	2.120	2.235	2.583	2.921	3.252	3.686	4.015
17	.689	.863	1.069	1.333	1.740	2.110	2.224	2.567	2.898	3.222	3.646	3.965
18	.688	.862	1.067	1.330	1.734	2.101	2.214	2.552	2.878	3.197	3.611	3.922
19	.688	.861	1.066	1.328	1.729	2.093	2.205	2.539	2.861	3.174	3.579	3.883
20	.687	.860	1.064	1.325	1.725	2.086	2.197	2.528	2.845	3.153	3.552	3.850
21	.686	.859	1.063	1.323	1.721	2.080	2.189	2.518	2.831	3.135	3.527	3.819
22	.686	.858	1.061	1.321	1.717	2.074	2.183	2.508	2.819	3.119	3.505	3.792
23	.685	.858	1.060	1.319	1.714	2.069	2.177	2.500	2.807	3.104	3.485	3.768
24	.685	.857	1.059	1.318	1.711	2.064	2.172	2.492	2.797	3.091	3.467	3.745
25	.684	.856	1.058	1.316	1.708	2.060	2.167	2.485	2.787	3.078	3.450	3.725
26	.684	.856	1.058	1.315	1.706	2.056	2.162	2.479	2.779	3.067	3.435	3.707
27	.684	.855	1.057	1.314	1.703	2.052	2.158	2.473	2.771	3.057	3.421	3.690
28	.683	.855	1.056	1.313	1.701	2.048	2.154	2.467	2.763	3.047	3.408	3.674
29	.683	.854	1.055	1.311	1.699	2.045	2.150	2.462	2.756	3.038	3.396	3.659
30	.683	.854	1.055	1.310	1.697	2.042	2.147	2.457	2.750	3.030	3.385	3.646
40	.681	.851	1.050	1.303	1.684	2.021	2.123	2.423	2.704	2.971	3.307	3.551
50	.679	.849	1.047	1.299	1.676	2.009	2.109	2.403	2.678	2.937	3.261	3.496
60	.679	.848	1.045	1.296	1.671	2.000	2.099	2.390	2.660	2.915	3.232	3.460
80	.678	.846	1.043	1.292	1.664	1.990	2.088	2.374	2.639	2.887	3.195	3.416
100	.677	.845	1.042	1.290	1.660	1.984	2.081	2.364	2.626	2.871	3.174	3.390
1000	.675	.842	1.037	1.282	1.646	1.962	2.056	2.330	2.581	2.813	3.098	3.300
∞	.674	.841	1.036	1.282	1.645	1.960	2.054	2.326	2.576	2.807	3.091	3.291
	50%	60%	70%	80%	90%	95%	96%	98%	99%	99.5%	99.8%	99.9%
	Confidence level *C*											

STANDARD STATISTICAL TABLES

1. Areas under the Normal Distribution

The table gives the cumulative probability up to the standardised normal value z i.e.

$$P[\,Z < z\,] = \int_{-\infty}^{z} \frac{1}{\sqrt{2\pi}} \exp(-\tfrac{1}{2}Z^2)\, dZ$$



z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5159	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7854
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8804	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9773	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9865	0.9868	0.9871	0.9874	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9924	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9980	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986

z	3.00	3.10	3.20	3.30	3.40	3.50	3.60	3.70	3.80	3.90
P	0.9986	0.9990	0.9993	0.9995	0.9997	0.9998	0.9998	0.9999	0.9999	1.0000