AFRICA UNIVERSITY

(A United Methodist-Related Institution)

"Investing in Africa's Future"

# FACULTY OF MANAGEMENT AND ADMINISTRATION

**COURSE TITLE:** **MMS409 Operations Research**

**SEMESTER 1:** **Final Examination - November 2016 (Mutare Parallel)**

**LECTURER:** **Dr S. Murairwa**

**TIME:** **3 Hours**

## INSTRUCTIONS

Answer **all** questions.

Start **each** question on a new page in your answer booklet.

The marks allocated to **each** question are shown at the end of the question.

**Show all your workings.**

Credit will be given for logical, systematic and neat presentations.

1. Suppose that a company product has a constant annual demand of 12 000 units. A unit costs \$2.50. The ordering cost is \$25 and the holding cost is charged at 20% of the cost per unit. There are 250 days per year and the lead time is 5 days. Calculate the following aspects of the company's inventory policy:
   a) Economic order quantity **[2 Marks]**
   b) Reorder point **[2 Marks]**
   c) Cycle time **[2 Marks]**
   d) Total annual cost **[2 Marks]**
   e) State four benefits of controlling inventory **[4 Marks]**

2. Develop a simple simulation model. With an example, explain the four variables of your simulation model **[8 Marks]**

3. State and explain four decision making situations **[8 Marks]**

4. A person requires 10, 12 and 12 units of chemicals A, B and C respectively for his garden. A liquid product contains 5, 2 and 1 units of A, B and C respectively per jar. A dry product contains 1, 2 and 4 units of A, B and C per carton. The liquid and dry products sell for \$3 per jar and \$2 per carton respectively.
   a) Formulate the linear programming model **[4 Marks]**
   b) How many each should be purchased in order to reduce or minimise cost and meet the requirements? Use the primal simplex method **[8 Marks]**
   c) Write the dual model of the linear programming model you formulated in (a) and state four useful aspects of duality **[7 Marks]**

5. The following table shows the activities, times and costs for a house project:

| | | Time (days) | | |
|---|---|---|---|---|
| **Activity** | **Preceding** | a | m | b |
| **A** | - | 3 | 5 | 6 |
| **B** | - | 2 | 4 | 6 |
| **C** | **A, B** | 5 | 6 | 7 |
| **D** | **A, B** | 7 | 9 | 10 |
| **E** | **B** | 2 | 4 | 6 |
| **F** | **C** | 1 | 2 | 3 |
| **G** | **D** | 5 | 8 | 10 |
| **H** | **D, F** | 6 | 8 | 10 |
| **I** | **E, G, H** | 3 | 4 | 5 |

   a) Determine the description of each activity **[5 Marks]**
   b) Draw the network diagram and determine the critical path **[9 Marks]**
   c) Can activity D be delayed? If so, by how many weeks? **[3 Marks]**
   d) Determine the earliest start and latest start times of the project. Analyse the slack column **[8 Marks]**
   e) Assuming the project completion time follows a normal distribution, what is the probability that the project can be completed in 25 days or less? **[6 Marks]**

6. If the customer arrival rate is 24 customers per hour and the service rate is 30 customers per hour.

a) Determine

i) Probability of no arrival **[2 Marks]**

ii) Probability of n customers in the queuing system **[2 Marks]**

iii) Average number of customers in the queuing system **[2 Marks]**

iv) Average number of customers in the waiting line **[2 Marks]**

v) Average time a customer spends in the total queuing system **[2 Marks]**

vi) Average time a customer spends waiting in the queue to be served **[2 Marks]**

vii) Probability that the server is idle **[3 Marks]**

b) Derive the distribution of the number of arrivals during a period t given the inter-arrival time is exponential with mean $\frac{1}{\lambda}$ **[8 Marks]**

**End of paper**

## ADDITIONAL INFORMATION

1. $K = \frac{C_c - C_n}{M}$

2. $M = T_n - T_c$

3. Let:
o = optimistic time estimate
m = most likely time estimate
p = pessimistic time estimate

$Mean\ (Expected\ Time): t = \frac{o+4m+p}{6}$

$Variance: \sigma^2 = \left(\frac{p-o}{6}\right)^2$

4. Social cost = Private costs + Negative Externalities

5. Social benefit = Private benefits + Positive externalities

6. $Z = \frac{x-\mu}{\sigma}$

7. $TC = \frac{1}{2}QC_h + \frac{DC_o}{Q} + DC$

8. $TC = \frac{1}{2}\left(1 - \frac{D}{P}\right)QC_h + \frac{DC_o}{Q} + DC$

9. $Q = \sqrt{\frac{2DC_o}{\left(1-\frac{D}{P}\right)C_h}}$

10. $Q = \sqrt{\frac{2DC_o}{C_h}}$

11. $f(t) = \lambda e^{-\lambda t},\ t > 0$

12. Queuing theory

- Single server model (MM/1)

$P_0 = 1 - \frac{\lambda}{\mu}$

$P_n = \left(\frac{\lambda}{\mu}\right)^n . P_0$

$L = \frac{\lambda}{\mu - \lambda}$

$$L_q = \frac{\lambda^2}{\mu(\mu - \lambda)}$$
$$W = \frac{1}{\mu - \lambda}$$
$$W_q = \frac{\lambda}{\mu(\mu - \lambda)}$$
$$U = \frac{\lambda}{\mu}$$
$$I = 1 - U$$

- Undefined and constant service time

$$P_0 = 1 - \frac{\lambda}{\mu}$$
$$L = L_q + \frac{\lambda}{\mu}$$
$$L_q = \frac{\lambda^2 \sigma^2 + \left(\frac{\lambda}{\mu}\right)^2}{2\left(1 - \frac{\lambda}{\mu}\right)}$$
$$W_q = \frac{L_q}{\lambda}$$
$$W = W_q + \frac{1}{\mu}$$
$$U = \frac{\lambda}{\mu}$$
$$I = 1 - U$$

- Finite calling population MM/1

$$P_0 = \frac{1}{\sum \frac{N!}{(N-n)!}\left(\frac{\lambda}{\mu}\right)^n}$$
$$P_n = \sum \frac{N!}{(N-n)!}\left(\frac{\lambda}{\mu}\right)^n P_0$$
$$L_q = N - \left(\frac{\lambda + \mu}{\lambda}\right)(1 - P_0)$$
$$L = L_q + (1 - P_0)$$
$$W_q = \frac{L_q}{(N - L)\lambda}$$
$$W = W_q + \frac{1}{\mu}$$

- Multiple server waiting line (MM/S)

$$P_0 = \frac{1}{\left[\sum_{n=0}^{n=c-1} \frac{1}{(n)!}\left(\frac{\lambda}{\mu}\right)^n\right] + \frac{1}{c!}\left(\frac{\lambda}{\mu}\right)^n \left(\frac{c\mu}{c\mu - \lambda}\right)}$$

$$P_n = \frac{1}{c!\, c^{n-c}}\left(\frac{\lambda}{\mu}\right)^n P_0, n > c; P_n = \frac{1}{n}\left(\frac{\lambda}{\mu}\right)^n P_0\,, n \leq c$$

$$L = \frac{\lambda\mu\left(\frac{\lambda}{\mu}\right)^c}{(c-1)!\,(c\mu - \lambda)^2} P_0 + \frac{\lambda}{\mu}$$

$$L_q = L - \frac{\lambda}{\mu}$$

$$W = \frac{1}{\lambda}$$

$$W_q = W - \frac{1}{\lambda}$$

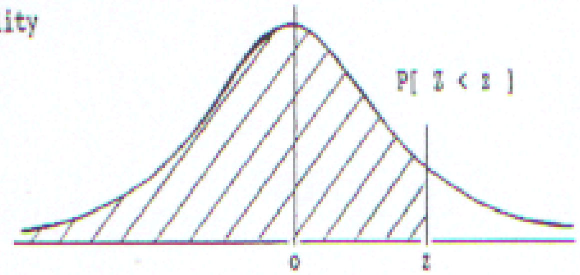
$$P_w = \frac{1}{c!}\left(\frac{\lambda}{\mu}\right)^c \left(\frac{c\mu}{c\mu - \lambda}\right) P_0$$

# STANDARD STATISTICAL TABLES

## 1. Areas under the Normal Distribution

The table gives the cumulative probability up to the standardised normal value z
i.e.

$$P[ Z < z ] = \int_{-\infty}^{z} \frac{1}{\sqrt{2\pi}} \exp(-\tfrac{1}{2}Z^2)\, dZ$$




| z | 0.00 | 0.01 | 0.02 | 0.03 | 0.04 | 0.05 | 0.06 | 0.07 | 0.08 | 0.09 |
|---|---|---|---|---|---|---|---|---|---|---|
| 0.0 | 0.5000 | 0.5040 | 0.5080 | 0.5120 | 0.5159 | 0.5199 | 0.5239 | 0.5279 | 0.5319 | 0.5359 |
| 0.1 | 0.5398 | 0.5438 | 0.5478 | 0.5517 | 0.5557 | 0.5596 | 0.5636 | 0.5675 | 0.5714 | 0.5753 |
| 0.2 | 0.5793 | 0.5832 | 0.5871 | 0.5910 | 0.5948 | 0.5987 | 0.6026 | 0.6064 | 0.6103 | 0.6141 |
| 0.3 | 0.6179 | 0.6217 | 0.6255 | 0.6293 | 0.6331 | 0.6368 | 0.6406 | 0.6443 | 0.6480 | 0.6517 |
| 0.4 | 0.6554 | 0.6591 | 0.6628 | 0.6664 | 0.6700 | 0.6736 | 0.6772 | 0.6808 | 0.6844 | 0.6879 |
| 0.5 | 0.6915 | 0.6950 | 0.6985 | 0.7019 | 0.7054 | 0.7088 | 0.7123 | 0.7157 | 0.7190 | 0.7224 |
| 0.6 | 0.7257 | 0.7291 | 0.7324 | 0.7357 | 0.7389 | 0.7422 | 0.7454 | 0.7486 | 0.7517 | 0.7549 |
| 0.7 | 0.7580 | 0.7611 | 0.7642 | 0.7673 | 0.7704 | 0.7734 | 0.7764 | 0.7794 | 0.7823 | 0.7854 |
| 0.8 | 0.7881 | 0.7910 | 0.7939 | 0.7967 | 0.7995 | 0.8023 | 0.8051 | 0.8078 | 0.8106 | 0.8133 |
| 0.9 | 0.8159 | 0.8186 | 0.8212 | 0.8238 | 0.8264 | 0.8289 | 0.8315 | 0.8340 | 0.8365 | 0.8389 |
| 1.0 | 0.8413 | 0.8438 | 0.8461 | 0.8485 | 0.8508 | 0.8531 | 0.8554 | 0.8577 | 0.8599 | 0.8621 |
| 1.1 | 0.8643 | 0.8665 | 0.8686 | 0.8708 | 0.8729 | 0.8749 | 0.8770 | 0.8790 | 0.8804 | 0.8830 |
| 1.2 | 0.8849 | 0.8869 | 0.8888 | 0.8907 | 0.8925 | 0.8944 | 0.8962 | 0.8980 | 0.8997 | 0.9015 |
| 1.3 | 0.9032 | 0.9049 | 0.9066 | 0.9082 | 0.9099 | 0.9115 | 0.9131 | 0.9147 | 0.9162 | 0.9177 |
| 1.4 | 0.9192 | 0.9207 | 0.9222 | 0.9236 | 0.9251 | 0.9265 | 0.9279 | 0.9292 | 0.9306 | 0.9319 |
| 1.5 | 0.9332 | 0.9345 | 0.9357 | 0.9370 | 0.9382 | 0.9394 | 0.9406 | 0.9418 | 0.9429 | 0.9441 |
| 1.6 | 0.9452 | 0.9463 | 0.9474 | 0.9484 | 0.9495 | 0.9505 | 0.9515 | 0.9525 | 0.9535 | 0.9545 |
| 1.7 | 0.9554 | 0.9564 | 0.9573 | 0.9582 | 0.9591 | 0.9599 | 0.9608 | 0.9616 | 0.9625 | 0.9633 |
| 1.8 | 0.9641 | 0.9649 | 0.9656 | 0.9664 | 0.9671 | 0.9678 | 0.9686 | 0.9693 | 0.9699 | 0.9706 |
| 1.9 | 0.9713 | 0.9719 | 0.9726 | 0.9732 | 0.9738 | 0.9744 | 0.9750 | 0.9756 | 0.9761 | 0.9767 |
| 2.0 | 0.9773 | 0.9778 | 0.9783 | 0.9788 | 0.9793 | 0.9798 | 0.9803 | 0.9808 | 0.9812 | 0.9817 |
| 2.1 | 0.9821 | 0.9826 | 0.9830 | 0.9834 | 0.9838 | 0.9842 | 0.9846 | 0.9850 | 0.9854 | 0.9857 |
| 2.2 | 0.9861 | 0.9865 | 0.9868 | 0.9871 | 0.9874 | 0.9878 | 0.9881 | 0.9884 | 0.9887 | 0.9890 |
| 2.3 | 0.9893 | 0.9896 | 0.9898 | 0.9901 | 0.9904 | 0.9906 | 0.9909 | 0.9911 | 0.9913 | 0.9916 |
| 2.4 | 0.9918 | 0.9920 | 0.9922 | 0.9924 | 0.9927 | 0.9929 | 0.9931 | 0.9932 | 0.9934 | 0.9936 |
| 2.5 | 0.9938 | 0.9940 | 0.9941 | 0.9943 | 0.9945 | 0.9946 | 0.9948 | 0.9949 | 0.9951 | 0.9952 |
| 2.6 | 0.9953 | 0.9955 | 0.9956 | 0.9957 | 0.9959 | 0.9960 | 0.9961 | 0.9962 | 0.9963 | 0.9964 |
| 2.7 | 0.9965 | 0.9966 | 0.9967 | 0.9968 | 0.9969 | 0.9970 | 0.9971 | 0.9972 | 0.9973 | 0.9974 |
| 2.8 | 0.9974 | 0.9975 | 0.9976 | 0.9977 | 0.9977 | 0.9978 | 0.9979 | 0.9980 | 0.9980 | 0.9981 |
| 2.9 | 0.9981 | 0.9982 | 0.9982 | 0.9983 | 0.9984 | 0.9984 | 0.9985 | 0.9985 | 0.9986 | 0.9986 |

| z | 3.00 | 3.10 | 3.20 | 3.30 | 3.40 | 3.50 | 3.60 | 3.70 | 3.80 | 3.90 |
|---|---|---|---|---|---|---|---|---|---|---|
| P | 0.9986 | 0.9990 | 0.9993 | 0.9995 | 0.9997 | 0.9998 | 0.9998 | 0.9999 | 0.9999 | 1.0000 |