"Investing in Africa's Future"

COLLEGE OF BUSINESS PEACE LEADERSHIP AND GOVERNANCE

MATHEMATICS FOR BUSINESS 1-MMS 101 -CONVENTIONAL

END OF SECOND SEMESTER EXAMINATIONS

JUNE 2017

LECTURER: Mr.T.Makambwa

DURATION: 3 HOURS

INSTRUCTIONS

Answer **ALL** the questions in **Section A** and any **Three questions from Section B** and each question has **20** marks. Total possible mark is **100.**

Start **each** question on a new page on your answer sheet.

The marks allocated to **each** question are shown at the end of the section.

Section A (40 Marks)

Answer ***all*** questions in this Section

1) Solve: $x = 2x - (6 - x)$ [3]

2) Solve: $\frac{6y}{7} = \frac{-3}{5}$ [2]

3) Solve: $\frac{3}{2}(4x - 3) = 2x - (4x - 3)$ [3]

4) Find the equation of the line with y-intercept 4 and slope – 2/3 [2]

5) Find a general linear equation of the line that passes through the points (4, -3) and (6, -7).[3]

6) Determine an equation of the vertical line that passes through the point (3, -6). [3]

7) Suppose f is a linear function such that $f(-2) = 5$ and $f(5) = 2$. Find $f(x)$. [4]

8) Suppose that a manufacturer will place 1000 units of a product on the market when the price is $10 per unit, and 1400 units when the price is $12 per unit. Find the supply equation for the product assuming the price p and quantity q are linearly related. [5]

9) Suppose the cost to produce 100 units of a product is $5000, and the cost to produce 125 units is $6000. If cost c is linearly related to output q, find an equation relating c and q. [5]

13) The demand per week for a new automobile is 400 units when the price is $16,700 each, and 500 units when the price is $14,900 each. Find the demand equation for the cars, assuming that it is linear. [5]

14) The demand function for an appliance company's line of washing machines is $p = 300 - 5q$, where p is the price (in dollars) per unit when q units are demanded (per week) by consumers. Find the level of production that will maximize the manufacturer's total revenue, and determine this revenue. [5]

Section B(60 Marks)

Answer any Three Questions from this Section

15. a) Given the demand function, $Q = 250 - 5P$ where Q is the number of children's watches demanded at £P each,

(i) Derive an expression for the point elasticity of demand in terms of P only. [4]

(ii) Calculate the point elasticity at each of the following prices, P =20; 25; 30.[8]

Describe the effect of price changes (expressed as % changes) on demand at each of these prices.

b) Given the demand function, $P = 60 - 0.2Q$, calculate the arc price elasticity of demand when:

(i) Price decreases from £50 to £40 and [4]

(ii) Price decreases from £20 to £10. [4]

16. a) The demand function for a perfectly competitive firm (same price charged for each good) is given as $P = 30$. The firm has fixed costs of £200 and variable costs of £5 per unit sold.

(i) Calculate the equilibrium quantity at the break-even point.

(ii) Calculate the value of total revenue and total cost at the break-even point.

b) The demand and supply functions for free-range Christmas turkeys are given by the equations: $P_d = 80 - 0.40d$ and $Ps. = 20 + 0.40S$

(i) Calculate the equilibrium price and quantity.

(ii) If the government provides a subsidy of £4 per bird:

(iii) Rewrite the equation of the supply function to include the subsidy.

(iv) Calculate the new equilibrium price and quantity.

(v) Outline the distribution of the subsidy, i.e. how much of the subsidy is received by the customer and by the supplier.

[20]

17. The demand and supply functions for a product (helicopter rides) are given by:

Demand function: $Q = 50 - 0.1P$

Supply function: $Q = -10 + 0.1P$

(a) Calculate the equilibrium price and quantity. Plot the demand and supply functions in the form $P = g(Q)$. Illustrate graphically the consumer and producer surplus at equilibrium.

(b) Calculate the consumer surplus at equilibrium.

(c) Calculate the producer surplus at equilibrium.

(d) Calculate the total surplus at equilibrium. [20]

18. A firm's total cost function is given by the equation $TC = 200 + 3Q$, while the demand function is given by the equation $P = 107 - 2Q$.

(a) Write down the equation of the total revenue function.

(b) Graph the total revenue function for $0 < Q < 60$. Hence, estimate the output, Q, and total revenue when total revenue is a maximum.

(c) Plot the total cost function on the diagram in (b). Estimate the break-even point from the graph. Confirm your answer algebraically.

(d) State the range of values for Q for which the company makes a profit.

[20]

19. Simple and Compound Interest

a) Suppose a person deposits $1000 in a savings account at the end of every six months. What is the value of the account at the end of five years if interest is at a rate of 10% compounded semiannually?

b) A person establishes the following retirement plan: an immediate deposit of $10,000 and quarterly payments of $1,500 at the end of each quarter into a savings account that earns 5% compounded quarterly, what is the amount of the investment after 21 years?

c) Suppose an annuity *due* consists of 6 yearly payments of $200 and the interest rate is 5% compounded annually. Determine (a) the present value and (b) the future value at the end of 6 years.

d) Suppose a corporation pays \$50,000 for a machine that has a useful life of eight years and a salvage value of \$5000. A sinking fund is established to replace the machine at the end of 8 years. The replacement machine will cost \$70,000. If equal payments are made into the fund at the end of every 6 months and the fund earns interest at the rate of 10% compounded semiannually, what should each payment be? [20]

20. Equations

a) Solve the following system algebraically: $3x - 4y = 18$ [3]

$$2x + 5y = -11$$

b) Solve for x: $\ln(x + 1) - \ln x = \ln 2$ [3]

c) Solve for x: $\log(x + 1) - \log(x - 2) = 1$ [4]

d) Solve for x: $(2 + x)^5 = 129.3$ [5]

e) Solve for x: $2^{\log 2x + \log 25} = 7$ [5]

END OF PAPER

❐ Financial mathematics

- **Amount due after t years (future value)**—bringing forward a single payment.

Simple interest: $P_t = P_0(1 + it)$

Compound interest (annual): $P_t = P_0(1 + i)^t$

Compound m times annually: $P_t = P_0\left(1 + \frac{i}{m}\right)^{mt}$

Continuous compounding: $P_t = P_0e^{it}$

- **Present value**—of a single payment due in t years from now.

Simple discounting: $P_0 = \frac{P_t}{1 + it} = P_t(1 + it)^{-1}$

Compound discounting: $P_0 = \frac{P_t}{(1 + i)^t} = P_t(1 + i)^{-t}$

Continuous discounting: $P_0 = P_te^{-it}$