"Investing in Africa's Future"

COLLEGE OF BUSINESS PEACE LEADERSHIP AND GOVERNANCE

MATHEMATICS FOR BUSINESS 2-MMS 105 -CONVENTIONAL

END OF SECOND SEMESTER EXAMINATIONS

APRIL/MAY 2017

LECTURER: Mr.T.Makambwa

DURATION: 3 HOURS

INSTRUCTIONS

Answer **ALL** the questions in **Section A** and any **Three questions from Section B** and each question has **20** marks. Total possible mark is **100.**

Start **each** question on a new page on your answer sheet.

The marks allocated to **each** question are shown at the end of the section.

Section A (40 Marks)

Answer ***all*** questions in this Section

Question One

A firm produces x tonnes of output at a total cost $C(x) = 0.1\ x^3 - 4x^2 + 20x + 5$ Find

(i) Average cost
(ii) Average Variable Cost
(iii) Average Fixed Cost
(iv) Marginal Cost
(v) Marginal Average Cost [10]

Question Two

The demand curve for a monopolist is given by x = 100--4p

(i) Find the total revenue, average revenue and marginal revenue.
(ii) At what value of x, the marginal revenue is equal to zero?

[10]

Question Three

a) If the demand function is $p = 35 - 2x - x^2$ and the demand $x0$ is 3, find the consumers' surplus. [6]

b) If the supply law is $p = 4 - x + x^2$, find the producers' surplus when the price is 6. [6]

c) The demand and supply law under a pure competition are given by $pd= 23 - x^2$ and $ps= 2x^2 - 4$. Find the consumers' surplus and producers' surplus at the market equilibrium price. [8]

Section B

Answer any ***three*** questions from this Section

Question Four

a) Given the demand function $q = (1{,}200 - 2p)^{0.5}$, what is elasticity of demand when quantity is 30?

b) Investigate the maxima and minima of the function $2x^3 + 3x^2 - 36x + 10$.

c) If the marginal revenue for a commodity is $MR = 9 - 6x^2 + 2x$, find the total revenue and demand function.

[6+7+7]

Question Five

A firm faces the demand schedule $p = 200 - 2q$ and the total cost function
$TC = \frac{2}{3}q^3 - 14q^2 + 222q + 50$
Derive expressions for the following functions and find out whether they have maximum or minimum points. If they do, say what value of q this occurs at and calculate the actual value of the function at this output.

(a) Marginal cost
(b) Average variable cost
(c) Average fixed cost
(d) Total revenue
(e) Marginal revenue
(f) Profit

[20]

Question Six

a) A School is preparing a trip for 400 students. The company who is providing the transportation has 10 buses of 50 seats each and 8 buses of 40 seats, but only has 9 drivers available. The rental cost for a large bus is $800 and $600 for the small bus. Calculate how many buses of each type should be used for the trip for the least possible cost.

b) A store wants to liquidate 200 of its shirts and 100 pairs of pants from last season. They have decided to put together two offers, A and B. Offer A is a package of one shirt and a pair of pants which will sell for $30. Offer B is a package of three shirts and a pair of pants, which will sell for $50. The store does not want to sell less than 20 packages of Offer A and less than 10 of Offer B. How many packages of each do they have to sell to maximize the money generated from the promotion?

[10+10]

Question Seven

a) The demand and supply functions under pure competition are $p_d = 16 - x^2$ and $p_s = 2x^2 + 4$. Find the consumers' surplus and producers' surplus at the market equilibrium price.

b) Solve the equations $2x + 8y + 5z = 5$, $x + y + z = --2$, $x + 2y --z = 2$ by using matrix method.

[10+10]

Question Eight

a) Find the general solution of the difference equation $Y_{t+1} - 0.95Y_t = 1000$

b) Find the particular solution, given $Y_5 = 20950$.

c) Determine whether the system will stabilize and if so ,what the stable value is .Plot the time to stability for t = 0 to 10 in steps of one

d) Solve the difference equation $3Y_{t+1} + 2Y_t = 44(0.8)^t$ given $Y_0 = 900$.

e) Show that the solution stabilizes and plot the time path to stability.

[20]

Question Nine

(a) Solve the differential equation $dy/dt = (1 - y)$.

(b) The rate at which an infection spreads in a poultry house is given by the equation $dP/dt = 0.75(2500 - P)$, where t is time in days. If P = 0 at t = 0,

(i) Solve the differential equation to determine an expression for the number of poultry (P) infected at any time t.

(ii) Graph the solution for t = 0 to 12. Hence describe the time path,

(i) Calculate the time taken for 1500 poultry to become infected. [20]

END OF PAPER

Mathematical Formulae

Integration Techniques

Many integration formulas can be derived directly from their corresponding derivative formulas, while other integration problems require more work. Some that require more work are substitution and change of variables, integration by parts, trigonometric integrals, and trigonometric substitutions.

Basic formulas

Most of the following basic formulas directly follow the differentiation rules.

1. $\int kf(x)\,dx = k\int f(x)\,dx$
2. $\int [f(x) \pm g(x)]\,dx = \int f(x)\,dx \pm \int g(x)\,dx$
3. $\int k\,dx = kx + C$
4. $\int x^n\,dx = \frac{x^{n+1}}{n+1} + C, n \neq -1$
5. $\int \sin x\,dx = -\cos x + C$
6. $\int \cos x\,dx = \sin x + C$
7. $\int \sec^2 x\,dx = \tan x + C$
8. $\int \csc^2 x\,dx = -\cot x + C$
9. $\int \sec x \tan x\,dx = \sec x + C$
10. $\int \csc x \cot x\,dx = -\csc x + C$
11. $\int e^x\,dx = e^x + C$
12. $\int a^x\,dx = \frac{a^x}{\ln a} + C, a > 0, a \neq 1$
13. $\int \frac{dx}{x} = \ln|x| + C$
14. $\int \tan x\,dx = -\ln|\cos x| + C$
15. $\int \cot x\,dx = \ln|\sin x| + C$
16. $\int \sec x\,dx = \ln|\sec x + \tan x| + C$
17. $\int \csc x\,dx = -\ln|\csc x + \cot x| + C$

18. $\int \frac{dx}{\sqrt{a^2 - x^2}} = \arcsin\frac{x}{a} + C$

19. $\int \frac{dx}{a^2 + x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$

20. $\int \frac{dx}{x\sqrt{x^2 - a^2}} = \frac{1}{a}\text{arcsec}\frac{x}{a} + C$

Applications of Integration

∴ Producer Surplus = $p_0q_0 - \int gx\,dx$ with limits from 0 to q_0

Consumer Surplus = $\int gx\,dx - p_0q_0$ with limits from 0 to q_0

Total Cost -∫ MCdx

Total Revenue =∫MRdx

Differentiation

The chain rule

If y is a function of u, which is itself a function of x, then

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$$

differentiate the outer function and multiply by the derivative of the inner function

The product rule

If $y = uv$ then = $\frac{dy}{dx} = u.\frac{dv}{dx} \quad x \quad v.\frac{du}{dx}$

This rule tells you how to differentiate the product of two functions: **multiply each function by the derivative of the other and add**

The quotient rule

If $y = \frac{u}{v}$ then $\frac{dy}{dx} = \frac{v.du/dx - u.dv/dx}{v^2}$

This rule tells you how to differentiate the quotient of two functions: **bottom times derivative of top, minus top times derivative of bottom, all over bottom squared**