AFRICA
UNIVERSITY
(A United Methodist-Related Institution)

"Investing in Africa's Future"

COLLEGE OF BUSINESS, PEACE, LEADERSHIP AND GOVERNANCE

MPM106 - STATISTICS FOR PUBLIC ADMINISTRATION

END OF SECOND SEMESTER FINAL EXAMINATION

APRIL/MAY 2017 (CONVENTIONAL)

LECTURER: DR S. MURAIRWA

TIME: 3 HOURS

INSTRUCTIONS

Answer **All** questions.

Start **each** question on a new page in your answer booklet.

The marks allocated are shown at the end of **each** question.

Show all your workings.

Credit will be given for logical, systematic and neat presentations.

1. The data obtained by measuring the age of 21 randomly selected students enrolled in Statistics at a university could be presented as the data list

18	18	19	20	22	20	18
19	19	18	18	21	18	19
18	24	19	18	20	17	17

Calculate
a) Mean **[2 marks]**
b) Median **[3 marks]**
c) Mode **[2 marks]**
d) Standard deviation **[4 marks]**

2. The bureau of Labour Statistics has sampled 30 communities in Zimbabwe and explained prices in each community at the beginning and end of August 2014 in order to find out approximately how the Consumer Price Index has change during the month. The percentage change in prices for the communities at the beginning of the month are given below:

0.8	0.2	-0.1	0.1	-0.2	0.2	0.3	0.5	-0.1	-0.2
0	0.6	0.3	0.2	1.0	-0.4	0	0.1	0.3	0.1
-0.5	-0.2	0	0.4	0.6	0	0.1	0.2	0.1	0.3

a) State the variable and determine whether it is categorical or numerical. If numerical, identify whether it is discrete or continuous. **[3 marks]**
b) Construct a frequency table for the data **[5 marks]**
c) Based on the table in (b), calculate:
   i) Mean **[2 marks]**
   ii) Mode **[3 marks]**
   iii) Median **[4 marks]**
   iv) Standard deviation **[3 marks]**
   v) Coefficient of skewness and interpret **[2 marks]**

3. Given the following events:

S = {list of number from 1 to 13}
P = {getting an odd number when tossing a fair die}
Q = {getting an even number when tossing a fair die}
R = {list of number from 2 to 12}
T = {list of prime number less than 12}

(a) Show the event in a Venn diagram **[4 marks]**
(b) Find the probability for
   i) $(S \cap P \cap R)$ **[2 marks]**
   ii) $(Q \cap R) \cap \overline{P}$ **[2 marks]**
   iii) $T \cup Q \cap \overline{R}$ **[2 marks]**

4. A sociologist wishes to estimate the proportion of all adults in a certain region who have never married. In a random sample of 1 320 adults, 145 have never married.
   (a) What is the population of interest? **[2 marks]**
   (b) What is the parameter of interest? **[2 marks]**
   (c) What is the statistic involved? **[2 marks]**

5. A student is interested to study how much the AU students spent the money in their campus life. She randomly selected students and asked them the amount of expenditures. Table below shows the expenditure of the students in last semester.

Month		January	February	March	April
Expenditure on	Books/study	50	45	50	60
	Life style	60	50	40	45

a) Construct the probability frequency table for the expenditure on books/study **[3 marks]**
b) Find the expected value of the expenditure on books/study **[2 marks]**

6. Seventeen percent of victims of financial fraud know the perpetrator of the fraud personally.
   (a) Construct the probability distribution for the number X of people in a random sample of five victims of financial fraud who knew the perpetrator personally **[6 marks]**
   (b) An investigator examines five cases of financial fraud every day. Find the most frequent number of cases each day in which the victim knew the perpetrator **[3 marks]**
   (c) An investigator examines five cases of financial fraud every day. Find the average number of cases per day in which the victim knew the perpetrator **[2 marks]**

7. In the past the average length of an outgoing telephone call from a business office has been 143 seconds. A manager wishes to check whether that average has decreased after the introduction of policy changes. A sample of 100 telephone calls produced a mean of 133 seconds, with a standard deviation of 35 seconds.
   (a) Perform the relevant test at the 1% level of significance **[6 marks]**
   (b) Verify your findings in (a) using p-value method **[5 marks]**
   (c) Construct a 99% confidence interval **[4 marks]**

8. Globally the long-term proportion of newborns who are male is 51.46%. A researcher believes that the proportion of boys at birth changes under severe economic conditions. To test this belief randomly selected birth records of 5,000 babies born during a period of economic recession were examined. It was found in the sample that 52.55% of the newborns were boys.
   (a) Determine whether there is sufficient evidence, at the 10% level of significance, to support the researcher's belief **[6 marks]**
   (b) Use an appropriate confidence interval to verify the results you obtained in (a) **[4 marks]**

9. Records of 40 used passenger cars and 40 used pickup trucks were randomly selected to investigate whether there was any difference in the mean time in years that they were kept by the original owner before being sold. For cars, the mean was 5.3 years with standard deviation of 2.2 years. For pickup trucks, the mean was 7.1 years with standard deviation of 3.0 years.
   (a) Construct the 95% confidence interval for the difference in the means **[4 marks]**
   (b) Test the hypothesis that there is a difference in the means against the null hypothesis that there is no difference. Use the 1% level of significance [6 marks]

End of paper

ADDITIONAL INFORMATION

1. Sturge's Rule:

   Number of class, $C = 1 + 3.3 \log n$

   Class width, $i > \frac{range}{C}$

2. Mean of grouped data $= \frac{\sum_{i=1}^{n} fx_i}{n}$

3. Mean of ungrouped data $= \frac{\sum_{i=1}^{n} x_i}{n}$

4. Mode $= L_{mo} + \left(\frac{\Delta_1}{\Delta_1 + \Delta_2}\right) i$

5 Median $= L_{me} + \left(\frac{\frac{n}{2} - F}{f_m}\right) i$

6. Standard deviation: $s = \sqrt{\frac{\sum_{i=1}^{n} fx_i^2 - \frac{\left(\sum_{n=1}^{n} fx_i\right)^2}{n}}{n-1}}$

7. Standard Deviation of ungrouped data: $s = \sqrt{\frac{\sum_{i=1}^{n} x_i^2 - \frac{\left(\sum_{i=1}^{n} x_i\right)^2}{n}}{n-1}}$

8. Coefficient of skewness: $S_k = \frac{3(mean - median)}{s} = \frac{mean - \text{mode}}{s}$

9. Conditional probability: $P(A \setminus B) = \frac{P(A \cap B)}{P(A)}$

10. Binomial Distribution
    - $P(X = x) = nC_x p^x q^{n-x}$

11. Poisson Distribution
    - $P(X = x) = \frac{e^{-\lambda}\lambda^x}{x!}$

12. Hypothesis testing (single mean)
    - $Z = \frac{\bar{X} - \mu}{\frac{\sigma}{\sqrt{n}}}$
    - $t = \frac{\bar{X} - \mu}{\frac{s}{\sqrt{n}}}$, $df = n - 1$

13. Hypothesis testing (single proportion)
    - $Z = \frac{p - \pi}{\sqrt{\frac{\pi(1-\pi)}{n}}}$

14. Hypothesis testing (difference of two means)

- $Z = \frac{\bar{X}_1 - \bar{X}_2}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$
- $t = \frac{\bar{X}_1 - \bar{X}_2}{\sqrt{\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}}}$, df = smaller $(n_1 - 1;\ n_2 - 1)$
- $t = \frac{\bar{X}_1 - \bar{X}_2}{\sqrt{\frac{S_p^2}{n_1} + \frac{S_p^2}{n_2}}}$

  Where $S_p^2 = \frac{S_1^2(n_1) + S_2^2(n_2)}{n_1 + n_2 - 2}$, $df = n_1 + n_2 - 2$
- $t = \frac{\bar{D} - \mu_D}{\frac{S_D}{\sqrt{n}}}$, df = n – 1

15. Hypothesis testing (difference of two proportions)
- $Z = \frac{p_1 - p_2}{\sqrt{\bar{p}\bar{q}\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$

16. Confidence Interval (Single mean)
- $\bar{X} - Z_{\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}} \leq \mu \leq \bar{X} + Z_{\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}}$
- $\bar{X} - t_{\frac{\alpha}{2}} \frac{S}{\sqrt{n}} \leq \mu \leq \bar{X} + t_{\frac{\alpha}{2}} \frac{S}{\sqrt{n}}$

17. Confidence Interval (Difference of two means)
- $(\bar{X}_1 - \bar{X}_2) - Z_{\frac{\alpha}{2}} \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}} \leq (\mu_1 - \mu_2) \leq (\bar{X}_1 - \bar{X}_2) + Z_{\frac{\alpha}{2}} \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$
- $(\bar{X}_1 - \bar{X}_2) - t_{\frac{\alpha}{2}} \sqrt{\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}} \leq (\mu_1 - \mu_2) \leq (\bar{X}_1 - \bar{X}_2) + t_{\frac{\alpha}{2}} \sqrt{\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}}$

  df = smaller $(n_1 - 1;\ n_2 - 1)$
- $(\bar{X}_1 - \bar{X}_2) - t_{\frac{\alpha}{2}} \sqrt{\frac{S_p^2}{n_1} + \frac{S_p^2}{n_2}} \leq (\mu_1 - \mu_2) \leq (\bar{X}_1 - \bar{X}_2) + t_{\frac{\alpha}{2}} \sqrt{\frac{S_p^2}{n_1} + \frac{S_p^2}{n_2}}$

  Where $S_p^2 = \frac{S_1^2(n_1) + S_2^2(n_2)}{n_1 + n_2 - 2}$, $df = n_1 + n_2 - 2$

18. Confidence Interval (Single proportion)
- $p - Z_{\frac{\alpha}{2}} \sqrt{\frac{pq}{n}} \leq \pi \leq p + Z_{\frac{\alpha}{2}} \sqrt{\frac{pq}{n}}$

19. Confidence Interval (Difference of two proportions)
- $(p_1 - p_2) - Z_{\frac{\alpha}{2}} \sqrt{\frac{p_1 q_1}{n_1} + \frac{p_2 q_2}{n_2}} \leq (\pi_1 - \mu_2) \leq (p_1 - p_2) + Z_{\frac{\alpha}{2}} \sqrt{\frac{p_1 q_1}{n_1} + \frac{p_2 q_2}{n_2}}$

20. $Z = \frac{X - \mu}{\sigma}$

21. Weighted Mean: $\bar{X}_w = \frac{\sum xw}{\sum w}$

22. $P(B_i / C) = \frac{P(C / B_i) P(B_i)}{\sum_{i=1}^{n} P(C / B_i) P(B_i)}$,

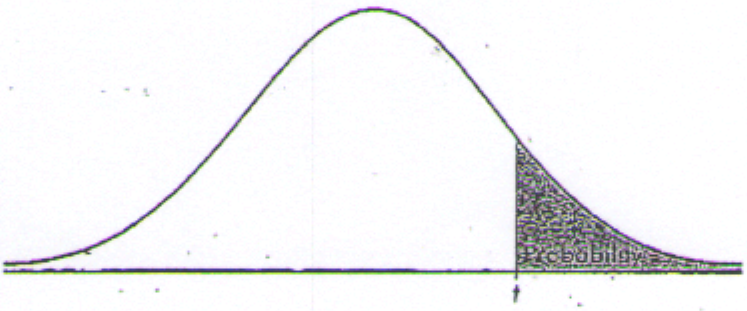


TABLE B: t-DISTRIBUTION CRITICAL VALUES

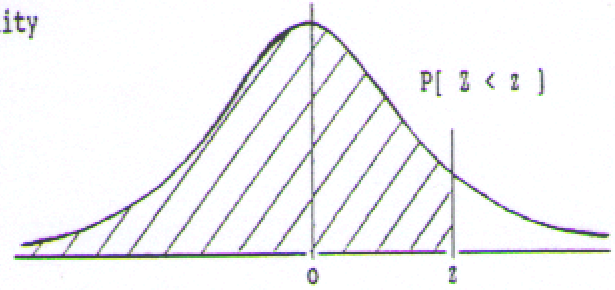
	Tail probability p											
df	.25	.20	.15	.10	.05	.025	.02	.01	.005	.0025	.001	.0005
1	1.000	1.376	1.963	3.078	6.314	12.71	15.89	31.82	63.66	127.3	318.3	636.6
2	.816	1.061	1.386	1.886	2.920	4.303	4.849	6.965	9.925	14.09	22.33	31.60
3	.765	.978	1.250	1.638	2.353	3.182	3.482	4.541	5.841	7.453	10.21	12.92
4	.741	.941	1.190	1.533	2.132	2.776	2.999	3.747	4.604	5.598	7.173	8.610
5	.727	.920	1.156	1.476	2.015	2.571	2.757	3.365	4.032	4.773	5.893	6.869
6	.718	.906	1.134	1.440	1.943	2.447	2.612	3.143	3.707	4.317	5.208	5.959
7	.711	.896	1.119	1.415	1.895	2.365	2.517	2.998	3.499	4.029	4.785	5.408
8	.706	.889	1.108	1.397	1.860	2.306	2.449	2.896	3.355	3.833	4.501	5.041
9	.703	.883	1.100	1.383	1.833	2.262	2.398	2.821	3.250	3.690	4.297	4.781
10	.700	.879	1.093	1.372	1.812	2.228	2.359	2.764	3.169	3.581	4.144	4.587
11	.697	.876	1.088	1.363	1.796	2.201	2.328	2.718	3.106	3.497	4.025	4.437
12	.695	.873	1.083	1.356	1.782	2.179	2.303	2.681	3.055	3.428	3.930	4.318
13	.694	.870	1.079	1.350	1.771	2.160	2.282	2.650	3.012	3.372	3.852	4.221
14	.692	.868	1.076	1.345	1.761	2.145	2.264	2.624	2.977	3.326	3.787	4.140
15	.691	.866	1.074	1.341	1.753	2.131	2.249	2.602	2.947	3.286	3.733	4.073
16	.690	.865	1.071	1.337	1.746	2.120	2.235	2.583	2.921	3.252	3.686	4.015
17	.689	.863	1.069	1.333	1.740	2.110	2.224	2.567	2.898	3.222	3.646	3.965
18	.688	.862	1.067	1.330	1.734	2.101	2.214	2.552	2.878	3.197	3.611	3.922
19	.688	.861	1.066	1.328	1.729	2.093	2.205	2.539	2.861	3.174	3.579	3.883
20	.687	.860	1.064	1.325	1.725	2.086	2.197	2.528	2.845	3.153	3.552	3.850
21	.686	.859	1.063	1.323	1.721	2.080	2.189	2.518	2.831	3.135	3.527	3.819
22	.686	.858	1.061	1.321	1.717	2.074	2.183	2.508	2.819	3.119	3.505	3.792
23	.685	.858	1.060	1.319	1.714	2.069	2.177	2.500	2.807	3.104	3.485	3.768
24	.685	.857	1.059	1.318	1.711	2.064	2.172	2.492	2.797	3.091	3.467	3.745
25	.684	.856	1.058	1.316	1.708	2.060	2.167	2.485	2.787	3.078	3.450	3.725
26	.684	.856	1.058	1.315	1.706	2.056	2.162	2.479	2.779	3.067	3.435	3.707
27	.684	.855	1.057	1.314	1.703	2.052	2.158	2.473	2.771	3.057	3.421	3.690
28	.683	.855	1.056	1.313	1.701	2.048	2.154	2.467	2.763	3.047	3.408	3.674
29	.683	.854	1.055	1.311	1.699	2.045	2.150	2.462	2.756	3.038	3.396	3.659
30	.683	.854	1.055	1.310	1.697	2.042	2.147	2.457	2.750	3.030	3.385	3.646
40	.681	.851	1.050	1.303	1.684	2.021	2.123	2.423	2.704	2.971	3.307	3.551
50	.679	.849	1.047	1.299	1.676	2.009	2.109	2.403	2.678	2.937	3.261	3.496
60	.679	.848	1.045	1.296	1.671	2.000	2.099	2.390	2.660	2.915	3.232	3.460
80	.678	.846	1.043	1.292	1.664	1.990	2.088	2.374	2.639	2.887	3.195	3.416
100	.677	.845	1.042	1.290	1.660	1.984	2.081	2.364	2.626	2.871	3.174	3.390
1000	.675	.842	1.037	1.282	1.646	1.962	2.056	2.330	2.581	2.813	3.098	3.300
∞	.674	.841	1.036	1.282	1.645	1.960	2.054	2.326	2.576	2.807	3.091	3.291
	50%	60%	70%	80%	90%	95%	96%	98%	99%	99.5%	99.8%	99.9%
	Confidence level C											

STANDARD STATISTICAL TABLES

1. Areas under the Normal Distribution

The table gives the cumulative probability up to the standardised normal value z i.e.

$$P[\, Z < z \,] = \int_{-\infty}^{z} \frac{1}{\sqrt{2\pi}} \exp(-\tfrac{1}{2}Z^2)\, dZ$$



z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5159	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7854
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8804	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9773	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9865	0.9868	0.9871	0.9874	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9924	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9980	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986

z	3.00	3.10	3.20	3.30	3.40	3.50	3.60	3.70	3.80	3.90
P	0.9986	0.9990	0.9993	0.9995	0.9997	0.9998	0.9998	0.9999	0.9999	1.0000