"Investing in Africa's Future"

COLLEGE OF BUSINESS PEACE LEADERSHIP AND GOVERNANCE

NCSC 101: DISCRETE STRUCTURES

END OF SECOND SEMESTER EXAMINATIONS

APRIL/MAY 2023

LECTURER: Mr. Timothy Makambwa

DURATION: 3 HOURS

INSTRUCTIONS

Answer **ALL** the questions in **Section A** and any Three questions from Section B and each question has **20** marks. Total possible mark is **100.**

Start **each** question on a new page on your answer sheet.

The marks allocated to **each** question are shown at the end of the section.

Section A – (Compulsory 40 Marks)

Question One

Prove that: $(p \rightarrow r) \vee (q \rightarrow r) \equiv (p \wedge q) \rightarrow r$ .Using the truth table below

p	q	r	$p \rightarrow r$	$q \rightarrow r$	$(p \rightarrow r) \vee (q \rightarrow r)$	$p \wedge q$	$(p \wedge q) \rightarrow r$
0	0	0					
0	0	1					
0	1	0					
0	1	1					
1	0	0					
1	0	1					
1	1	0					
1	1	1					

[15]

Question Two

a) In the CSI Conference in Delhi, 500 delegates attended.200 of them could take tea, 350 could take coffee and 10 did not take either tea or coffee. Then answer the following questions:

i. How many can take both tea and coffee,
ii. How many can take tea only and
iii. How many can take coffee only.

[6]

b) In a group of 191 students, 10 are taking English, Computer Science and Music, 36 are English and Computer Science, 20 are taking English and Music. 18 are taking Computer Science and Music, 65 are taking English, 76 Computer Science and 63 are taking Music. Then answer the followings:

i. How many are taking English and Music but not Computer Science,
ii. How many are taking Computer Science and Music but not English,
iii. How many are taking Computer Science but neither English nor Music,
iv. How many are taking none of the Three subjects

[10]

Question Three

A real number between 0 and 1 is represented by negative powers of the base. For example, in decimal $0.125 = 1 \times 10^{-1} + 2 \times 10^{-2} + 5 \times 10^{-3}$.

Convert the following numbers to decimal.

a) 0.1101_2

b) 0.000001_2

c) 111.111_2

d) $0.A8_{16}$

e) 111.111_{16}

[5]

Question Four

Find the Greatest Common Divisor (GCD) of each pair using the Euclidean algorithm.
a) 275 and 115
b) 999 and 123
c) 456 and 144
d) 725 and 1000

[4]

Section B

Answer any *three* in this Section

Question Five

Prove the following formulas for all positive integers n.

a) $1 + 2 + 3 + 4 + 5 + \ldots + n = n(n + 1)/ 2$
b) $2 + 4 + 6 + 8 + 10 + \ldots + 2n = n^2 + n$
c) $1 + 2 + 4 + 8 + 16 + \ldots + 2^{n-1} = 2^n - 1$
d) $1 + 3 + 9 + 27 + 81 + \ldots + 3^{n-1} = (3^n - 1)/ 2$
e) $1 + 4 + 9 + 16 + 25 + \ldots + n^2 = n(n + 1)(2n + 1)/ 6$

[4x5]

Question Six

Prove by induction for all positive integers n.

a) $2^{2n} - 1$ is a multiple of 3
b) 7 is a divisor of $2^{3n} - 1$
c) $n^3 + 2n$ is a multiple of 3
d) $n^5 - n \bmod 5 = 0$
e) $2^{n+2} + 3^{2n+1}$ is a multiple of 7

[4x5]

Question Seven

Prove the following propositions.

a) $n < 2^2 \ \forall\ n \geq 1$

b) $2^n < n! \ \forall\ n \geq 4$

c) $3^n < n! \ \forall\ n \geq 7$

d) $2^n > n^n \ \forall\ n \geq 5$

e) $n! < n^n \ \forall\ n \geq 2$

[4x5]

Question Eight

Prove the following propositions using proof by contrapositive
a) If x^2 is even then x must be even.
b) If x^3 is even then x must be even.
c) If $x^2 - 2x$ is even then x must be even.
d) If $x^3 - 4x + 2$ is odd then x must be odd.

[4x5

Question 8

Prove: $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

a) Using the logical equivalence method
b) Using the Membership table

[20]

Question 9

a) $A = \{1,2,3\}$

R = {(1,2),(2,1)}

R = {(1,1),(2,2),(3,3)}

R = {(1,1),(2,2),(3,3),(1,2)} [10]

Find out whether the relations are symmetric, reflexive, or transitive

b) (a,b) ε R if a-b is a multiple of 3. Show that it is an equivalence relation i.e.it is reflexive, symmetric, and transitive [10]

END OF EXAMINATION