"Investing in Africa's Future"

COLLEGE OF BUSINESS, PEACE, LEADERSHIP, AND GOVERNANCE

NCSC 102: ALGEBRA

END OF SECOND SEMESTER EXAMINATIONS

APRIL/MAY 2023

LECTURER: MR A.C MUZENDA

DURATION: 3 HOURS

INSTRUCTIONS

Answer all Questions in Section A and **any three questions from Section B**
Total possible mark is 100

Start **each** question on a new page in your answer Booklet.

The marks allocated to **each** question are shown at the end of the section.

SECTION A: ANSWER ALL QUESTIONS

QUESTION 1

a. Given the function f(x) = 10x + x + 25 + 20(x + 3)

i. Find the value of f (6) [2]

ii. If f(x) = 240, what is the value of x? [3]

b. Given B = $\begin{pmatrix} 2 & X-2 & 1 \\ 1 & 2 & -1 \\ 3 & 4+X & -2 \end{pmatrix}$ Find X if det(B) = 6, hence find B^{-1} [7]

c. Given that vectors **a** and **b**, are not parallel. Simplify fully the following expression

(2a+b) ∧ (a-2b) [6]

d. The 2× 2 matrices A, B and C are given below in terms of the scalar constants x.

A= $\begin{pmatrix} 2 & x \\ 3 & 1 \end{pmatrix}$ **B**= $\begin{pmatrix} 4 & 1 \\ 3 & 2 \end{pmatrix}$ **C** = $\begin{pmatrix} 3x+2 & 7 \\ -2x+3 & 4x \end{pmatrix}$

i. Find an expression for AB, in terms of x. [5]

ii. Determine the value of x, given $\mathbf{B}^{\mathbf{T}}\mathbf{A}^{\mathbf{T}} = \mathbf{C}$ [5]

e. The position vectors of the points **A, B** and **C** are given below

OA = -**i** + 2**j** +2**k**, OB = 3**i** + 4**j** – **k** and **OC** = **i** + 4**j** +**k**.

i. Show that **OA**, **OB** and **OC** are linearly dependent. [6]

ii. Find the area of the triangle **ABC** [6]

SECTION B

Answer any three questions

QUESTION 2

a. The following vectors are given.

$$\mathbf{a} = 2\mathbf{i} + 3\mathbf{j} - \mathbf{k}$$
$$\mathbf{b} = \mathbf{i} + 2\mathbf{j} + \mathbf{k}$$
$$\mathbf{c} = \mathbf{j} + 3\mathbf{k}$$

i. Show the three vectors are coplanar. [5]

ii. Express **a** in terms of **b** and **c**. [6]

b. The 3 × 3matrix A is given below.

$$x + 3y + 5z = 6$$
$$6x - 8y + 4z = -3$$
$$3x + 11y + 13z = 17$$

Solve the equation using Crammer's rule [9]

QUESTION 3

a. The 3 × 3matrix A is given below.

$$x + 3y + 2z = 13$$
$$3x + 2y - z = 4$$
$$2x + y + z = 7$$

Solve equation using Gaussian Elimination method [10]

b. The vectors **a** and **b** are such so that

$$|\mathbf{a}| = 10,\ |\mathbf{b}| = 10 \text{ and } \mathbf{a}.\mathbf{b} = 30.$$

Prove that $|\mathbf{a} \wedge \mathbf{b}| = 10$ [8]

c. Explain a sub space in relation to vectors. [2]

QUESTION 4

a. Two non-zero vectors **a** and **b** have respective magnitudes a and b, respectively.

Given that $f = |\mathbf{a} \wedge \mathbf{b}|$ and $g = |\mathbf{a}.\mathbf{b}|$, show that

$$f^2 + g^2 = a^{2+} b^2$$ [10]

b. Find, in terms of x the inverse of the following 2× 2 matrix.

$$\begin{pmatrix} 2x+3 & 4x \\ -x+3 & 2x \end{pmatrix}$$ [5]

c. The distinct square matrices **A** and **B** have the properties

$\mathbf{AB} = \mathbf{B}^5\mathbf{A}$ and $\mathbf{B}^6 = \mathbf{I}$ where **I** is the identity matrix.

Prove that **BAB** = **A** **[5]**

QUESTION 5

a. x +y -2z =2
3x -y + 6z = 2
6x +5y -9z = k

i. Show that the system of equations does not have a unique solution. [3]
ii. Show that there exists a value of *k* for which the system is consistent. [3]
iii. Show, by reducing the system into row echelon form, that the consistent solution of the system can be written as

$$x = 1 - t,\ y = 3t + 1,\ z = t$$

where *t* is a scalar parameter. [4]

b. Relative to a fixed origin O, the respective position vectors of three points A, B and C are

(3, 2, 9), (-5, 11, 6) and (4, 0, -8)

i. Determine, in component form, the vectors **AB** and **AC**. [3]
ii. Hence find, to the nearest degree, the angle **BAC**. [3]
iii. Calculate the area of the triangle **BAC**. [4]

THE END