AFRICA UNIVERSITY
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COLLEGE OF BUSINESS, PEACE, LEADERSHIP, AND GOVERNANCE

NMMS 105: MATHEMATICS FOR BUSINESS 2

END OF SECOND SEMESTER FINAL EXAMINATION

APRIL/MAY 2023

LECTURER: MUGWAGWA T.M

DURATION: 3 HOURS

INSTRUCTIONS

Answer **ALL** the questions in **Section A** and any **Three questions from Section B** and each question has **20** marks. Total possible mark is **100.**

Start **each** question on a new page on your answer sheet.

The marks allocated to **each** question are shown at the end of the section.

Section A (40 Marks)

Answer ***all*** questions in this Section

Question One

Differentiate the following

a) $y = (3x - 4)^3$
b) $y = (x^2 + 3x + 5)^5$
c) $y = x^2(2x + 1)^5$
d) $y = x^3\sqrt{(2x+3)}$
e) $y = \frac{x^2}{x^2+4}$

[15]

Question Two

Integrate the following:

a) $\int 2x dx$
b) $\int \sqrt{x}\, dx$
c) $\int e^{2x} dx$
d) $\int \frac{0.5}{x} dx$
e) $\int (65x+3^x)\, dx$ **[15]**

Question Three

a) A firm faces the demand schedule $p = 184 - 4q$ and the TC function TC = $q^3 - 21q^2 + 160q + 40$. What output will maximize profit? **[10]**

Section B

Answer any three questions from this Section

Question Four

(a)A firm produces x tonnes of output at a total cost

$$C(x) = \frac{x^3}{10} - 4x^2 + 20x + 5$$

Find

(i) Average cost [2]

(ii) Average Variable Cost [2]

(iii) Average Fixed Cost [2]

(iv) Marginal Cost and [3]

(v) Marginal Average Cost. [3]

b) The demand curve for a monopolist is given by $x = 100-4p$

(i) Find the total revenue, average revenue and marginal revenue.

(ii) At what value of x, the marginal revenue is equal to zero? [8]

Question Five

Find the minimum and maximum of the objective function subject to the constraints

a) Objective function: 4x+5y

Constraints

$x \geq 0$

$y \geq 0$

$x+6y \leq 6$ [4]

b) Objective function: $3x+2y$

Constraints

$x \geq 0$

$y \geq 0$

$x+3y \leq 15$

$4x+y \leq 16$ [8]

c) Objective function: $6x+7y$

Constraints

$x \geq 0$

$y \geq 0$

$4x+3y \geq 24$

$x+3y \geq 15$ [8]

Question Six

a) The marginal cost function of manufacturing x units of a commodity is $6 + 10x - 6x^2$. Find the total cost and average cost, given that the total cost of producing 1 unit is 15.

a) If the marginal revenue for a commodity is MR = $9 - 6x2 + 2x$, find the total revenue and demand function.

b) The marginal cost function of manufacturing x units of a commodity is $3 - 2x – x^2$. If the fixed cost is 200, find the total cost and average cost functions.

[6+7+7]

Question Seven

The demand and supply functions for a good are $P_d = 100 - 0.5\,Q$ and $P_s = 10 + 0.5Q$, respectively.

(a) Calculate the equilibrium price and quantity; confirm your answer graphically.

(b) Calculate consumer and producer surplus at equilibrium.

[12+8]

Question Eight

a) Solve and find a general solution of the equation $y^1e^{-x}+e^{2x} = 0$

b) Solve the differential equation $y^1 = 12e^{0.6t}$, given y=80 when t=0

i) find the particular solution of $y^1 + 2y = 6$, given y=1 when x=0

[20]

Question Nine

Solve each of the following difference equations for the indicated variable by the iteration method.

(a) $Y_{t+1} - 0.8\,Y_t = 10$, given $Y_1 = 5$. Find Y_5.

(b) $P_{t+2} = 4P_{t+1} - 8P_t$ given $P_1 = 20$, $P_2 = 18$. Find P_5.

(c) $P_t = 0.6P_{t-1} + 80$, given $P_1 = 100$. Find P_5

[6+7+7]

Question Ten

a) Solve using matrices the equations $2x-y = 3$, $5x+y = 4$.

b) Solve the equations $2x + 8y + 5z = 5$, $x + y + z = -2$, $x + 2y - z = 2$ by using the matrix method or otherwise.

[6+14]

END OF EXAMINATION

Mathematical Formulae

Integration Techniques

Many integration formulas can be derived directly from their corresponding derivative formulas, while other integration problems require more work. Some that require more work are substitution and change of variables, integration by parts, trigonometric integrals, and trigonometric substitutions.

Basic formulas

Most of the following basic formulas directly follow the differentiation rules.

1. $\int kf(x)\,dx = k\int f(x)\,dx$
2. $\int [f(x) \pm g(x)]\,dx = \int f(x)\,dx \pm \int g(x)\,dx$
3. $\int k\,dx = kx + C$
4. $\int x^n\,dx = \frac{x^{n+1}}{n+1} + C, n \neq -1$
5. $\int \sin x\,dx = -\cos x + C$
6. $\int \cos x\,dx = \sin x + C$
7. $\int \sec^2 x\,dx = \tan x + C$
8. $\int \csc^2 x\,dx = -\cot x + C$
9. $\int \sec x \tan x\,dx = \sec x + C$
10. $\int \csc x \cot x\,dx = -\csc x + C$
11. $\int e^x\,dx = e^x + C$
12. $\int a^x\,dx = \frac{a^x}{\ln a} + C, a > 0, a \neq 1$
13. $\int \frac{dx}{x} = \ln|x| + C$
14. $\int \tan x\,dx = -\ln|\cos x| + C$
15. $\int \cot x\,dx = \ln|\sin x| + C$
16. $\int \sec x\,dx = \ln|\sec x + \tan x| + C$
17. $\int \csc x\,dx = -\ln|\csc x + \cot x| + C$

18. $\int \frac{dx}{\sqrt{a^2 - x^2}} = \arcsin\frac{x}{a} + C$

19. $\int \frac{dx}{a^2 + x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$

20. $\int \frac{dx}{x\sqrt{x^2 - a^2}} = \frac{1}{a}\text{arcsec}\frac{x}{a} + C$

Applications of Integration

$\therefore$ Producer Surplus = $p_0q_0 - \int gx\, dx$ with limits from 0 to q_0

Consumer Surplus = $\int gx\, dx - p_0q_0$ with limits from 0 to q_0

Total Cost = $\int$ MC dx

Total Revenue = $\int$ MR dx

Differentiation

The chain rules

If *y* is a function of *u*, which is itself a function of *x*, then

$$\frac{dy}{dx} = \frac{dy}{du} \text{ x } \frac{du}{dx}$$

differentiate the outer function and multiply by the derivative of the inner function

The product rule

If *y=uv*, then $\frac{dy}{dx} = u.\frac{dv}{dx} \text{ x } v.\frac{du}{dx}$

This rule tells you how to differentiate the product of two functions: **multiply each function by the derivative of the other and add**

The quotient rule

If $y = \frac{u}{v}$ then $\frac{dy}{dx} = \frac{v.du/dx - u.dv/dx}{v^2}$

This rule tells you how to differentiate the quotient of two functions: **bottom times derivative of top, minus top times derivative of bottom, all over bottom squared**