**"Investing in Africa's Future"**

**COLLEGE OF BUSINESS, PEACE, LEADERSHIP AND GOVERNANCE**

# NMMS409: OPERATIONS RESEARCH

**END OF SEMESTER EXAMINATION**

**APRIL/MAY 2023**

**PROF S. MURAIRWA**

**3 HOURS**

## INSTRUCTIONS

Answer **All** questions in Section A and **any three** questions in Section B.

Start **each** question on a new page in your answer booklet.

The marks allocated to **each** question are shown at the end of the question.

**Show all your workings.**

Credit will be given for logical, systematic and neat presentations.

## SECTION A: ANSWER ALL QUESTIONS

1. Discuss the useful aspects of duality. [**4 marks**]

2. Typically, a simulation model will attempt to describe a business system by several equations. These equations are characterised by four types of variables. State and explain the four variables. [**8 marks**]

3. A company manufactures two types of products A and B and sells them at a profit of $2 on type A and $3 on type B. Each product is processed on two machines G and H. Type A requires one minute of processing on G and two minutes on H. Type B requires one minute on G and one minute on H. Machine G is available for not more than 6 hours 40 minutes while machine H is available for 10 hours during any working day.

   (a). Formulate the problem as a linear programming model. [**5 marks**]
   (b). Use the graphical method to solve the linear programming model. [**5 marks**]
   (c). Use the Simplex method to find the optimal tableau. [**12 marks**]
   (d). If the coefficients of the objective function are increased by 30%, determine the new basic variables and associated optimum profit. [**3 marks**]
   (e). Construct the Dual model of the Primal model. [**3 marks**]

## *S*ECTION B: ANSWER ANY THREE QUESTIONS

4. The activities to produce a new product are shown in the table below.

| Activity | Preceding | Time in days | | |
|---|---|---|---|---|
| | | a | m | b |
| **A** | **-** | 5 | 8 | 17 |
| **B** | **-** | 3 | 12 | 15 |
| **C** | **A** | 4 | 7 | 10 |
| **D** | **A** | 5 | 8 | 23 |
| **E** | **B, C** | 1 | 1 | 1 |
| **F** | **B, C** | 1 | 4 | 13 |
| **G** | **D, E** | 3 | 6 | 9 |
| **H** | **D, E** | 1 | 2.5 | 7 |
| **I** | **H** | 1 | 1 | 1 |
| **J** | **F, G** | 2 | 2 | 2 |
| **K** | **G, I** | 5 | 8 | 11 |

   (a) Draw and label a PERT/CPM network diagram for the project. **[5 marks]**
   (b) Calculate the earliest start and latest start times and slacks. Briefly discuss the slack results. **[10 marks]**
   (c) Assuming the production completion time follows a Gaussian distribution, what is the probability that the production can be completed in 40 days or less? **[5 marks]**

5. A transportation problem is presented below. Each cell represents a shipping route and the unit shipping cost is given in the upper right-hand box in each of the cells.

| | $d_1$ | $d_2$ | $d_3$ | Supply |
|---|---|---|---|---|
| $s_1$ | *15* | *30* | *20* | *50* |
| $s_2$ | *30* | *40* | *35* | *30* |
| Demand | *25* | *45* | *10* | |

a) Construct a network of the transportation problem. [**5 marks**]

b) Find the initial tableau of the transportation problem using the following methods:

   i. The Northwest corner [**5 marks**]

   ii. The minimum cost of the row [**5 marks**]

c) Formulate the linear programming problem. [**5 marks**]

6. Africa University is preparing a summer campsite in Inyanga. Africa University estimated that attendance can fall into one of four categories, 2 000; 2 500; 3 000, and 3 500 persons. The campsite cost will be minimised if built to meet the exact demand. Deviations above or below the ideal demand levels incur unnecessary additional costs resulting from building surplus (unused) capacity or lost income opportunities when the demand is not met. Let $a_1, a_2$, $a_3$, and $a_4$ represent the sizes of the campsites, and $s_1$, $s_2$, $s_3$, and $s_4$ represent the level of attendance. The cost matrix in thousands of dollars is given below:

| | | **States of Nature** | | | |
|---|---|---|---|---|---|
| | | **1** | **2** | **3** | **4** |
| **Sizes of the campsites** | **1** | 5 | 10 | 18 | 25 |
| | **2** | 8 | 7 | 12 | 23 |
| | **3** | 21 | 18 | 12 | 21 |
| | **4** | 31 | 22 | 19 | 15 |

Analyse and comment on the situation above using the following decision criteria:

(a) Laplace criterion. [**3 marks**]

(b) Minimax criterion. [**3 marks**]

(c) Savage regret criterion. [**5 marks**]

(d) Hurwicz criterion ($\alpha = 0.25$). [**5 marks**]

(e) Decision tree analysis with equal probability of occurrence. [**4 marks**]

7. Attempt the following questions:
   a) A textile shop has an estimated annual demand of 10 000 metres of cloth, an annual carrying cost of $0.75 per metre, and an ordering cost of $150 per order. State and explain five importance of inventory control in production. **[5 marks]**
   b) Patients arrive at a hospital at the rate of one every 12 minutes. The time between the arrival of patients follows an exponential distribution.
      i. With a diagram, explain the parameters of the queuing system. **[5 marks]**
      ii. Prove that the distribution of the number of arrival patients during a period *t* given that the inter-arrival time is exponential with mean $\frac{1}{\lambda}$ is a Poisson distribution. **[5 marks]**
   c) Discuss five characteristics of Operations Research. [**5 marks**]

**End of paper**

## ADDITIONAL INFORMATION

1. $K = \frac{C_c - C_n}{M}$

2. $M = T_n - T_c$

3. Let:
o = optimistic time estimate
m = most likely time estimate
p = pessimistic time estimate

$Mean\ (Expected\ Time): t = \frac{o+4m+p}{6}$
$Variance: \sigma^2 = \left(\frac{p-o}{6}\right)^2$

4. $Z = \frac{x-\mu}{\sigma}$

5. $TAC = \frac{Qc_h}{2} + \frac{D}{Q}c_o + DC$

6. $Q^* = \sqrt{\frac{2Dc_o}{c_h}}$

7. $TAC = DC + \frac{Dc_o}{Q} + \frac{1}{2}\left(1 - \frac{d}{p}\right)Qc_h$

8. $Q^* = \sqrt{\frac{2Dc_o}{\left(1-\frac{d}{p}\right)c_h}}$

9. $TAC = Dc + \frac{Dc_o}{Q} + \frac{(Q-S)^2}{2Q}c_h + \frac{S^2}{2Q}c_b$

10. $P_n'(t) = \frac{(\lambda t)^n e^{-\lambda t}}{n!}$