"Investing in Africa's future"

COLLEGE OF ENGINEERING AND APPLIED SCIENCES (CEAS)

NCSC 101- DISCRETE STRUCTURES

END OF FIRST SEMESTER EXAMINATIONS

NOVEMBER 2023

LECTURER: Mr. Timothy Makambwa

DURATION: 3 HOURS

INSTRUCTIONS

Answer **ALL** the questions in **Section A** and any **Three questions from Section B** and each question has **20** marks. Total possible mark is **100.**

Start **each** question on a new page on your answer sheet.

The marks allocated to **each** question are shown at the end of the section.

Section A (40 Marks)

Answer *all* questions in this Section

Question One

a) Fill out the following truth tables and determine which statements are tautologies, contradictions, or neither.

STATEMENT: $(p \wedge q) \vee \sim p$

p	q	$p \wedge q$	$\sim p$	$(p \wedge q) \vee \sim p$
T	T			
T	F			
F	T			
F	F			

[3]

b)

STATEMENT: $\sim p \rightarrow (\sim p \vee q)$

p	q	$\sim q$	$\sim p$	$\sim p \vee q$	$\sim q \rightarrow (\sim p \vee q)$

[4]

c)

STATEMENT: $(\sim p \to q) \lor (\sim p \land \sim q)$

p	q					

[4]

d)

STATEMENT: $[p \land (q \lor \sim r)] \to (\sim p \land q)$

Hint: list the nine columns left to right as follows

$p,\ q,\quad r,\quad \sim r,\quad q \lor \sim r,\quad p \land (q \lor \sim r),\ \sim p,\quad \sim p \land q,$
and finally $[p \land (q \lor \sim r)] \to (\sim p \land q)$

[11]

Question Two

a) In the CSI Conference in Delhi, 500 delegates attended.200 of them could take tea, 350 could take coffee and 10 did not take either tea or coffee. Then answer the following questions:

    i.   How many can take both tea and coffee, [2]
    ii.   How many can take tae only and [2]
    iii.   How many can take coffee only. [2]

b) In a group of 191 students, 10 are taking English, Computer Science and Music, 36 are English and Computer Science, 20 are taking English and Music. 18 are taking Computer Science and Music, 65 are taking English, 76 Computer Science and 63 are taking Music. Then answer the followings:

    i.   How many are taking English and Music but not Computer Science, [3]
    ii.   How many are taking Computer Science and Music but not English, [3]
    iii.   How many are taking Computer Science but neither English nor Music, [3]
    iv.   How many are taking none of the Three subjects [3]

Section B

Answer any *three* in this Section

Question Three

Prove the following formulas for all positive integers n.

a) $1 + 2 + 3 + 4 + 5 + \ldots + n = n(n + 1) \div 2$ [4]

b) $2 + 4 + 6 + 8 + 10 + \ldots + 2n = n^2 + n$ [4]

c) $1 + 2 + 4 + 8 + 16 + \ldots + 2^{n-1} = 2^n - 1$ [4]

d) $1 + 3 + 9 + 27 + 81 + \ldots + 3^{n-1} = (3^n - 1) \div 2$ [4]

e) $1 + 4 + 9 + 16 + 25 + \ldots + n^2 = n(n + 1)(2n + 1) \div 6$ [4]

Question Four

Prove by induction for all positive integers n.

a) $2^{2n} - 1$ is a multiple of 3 [4]
b) 7 is a divisor of $2^{3n} - 1$ [4]
c) $n^3 + 2n$ is a multiple of 3 [4]
d) $n^5 - n \bmod 5 = 0$ [4]

e) $2^{n+2} + 3^{2n+1}$ is a multiple of 7 [4]

Question Five

Identify each proposition as a tautology, contradiction, or contingency/satisfiable .

a) $(p \wedge q) \to p$ [4]

b) $p \to (p \vee q)$ [4]

c) $p \to (p \to q)$ [4]

d) $p \to (q \to p)$ [4]
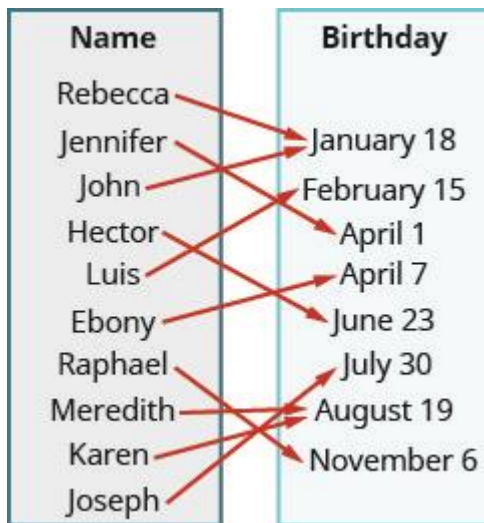
e) $\neg p \wedge \neg(p \to q)$ [4]

Question Six

a) In the following exercises, for each relation **a.** find the domain of the relation **b.** find the range of the relation.

1. $\{(1,4),(2,8),(3,12),(4,16),(5,20)\}\{(1,4),(2,8),(3,12),(4,16),(5,20)\}$

2. $\{(1,-2),(2,-4),(3,-6),(4,-8),(5,-10)\}\{(1,-2),(2,-4),(3,-6),(4,-8),(5,-10)\}$

3. $\{(1,7),(5,3),(7,9),(-2,-3),(-2,8)\}\{(1,7),(5,3),(7,9),(-2,-3),(-2,8)\}$

4. $\{(11,3),(-2,-7),(4,-8),(4,17),(-6,9)\}\{(11,3),(-2,-7),(4,-8),(4,17),(-6,9)\}$

b) In the following exercises, use the mapping of the relation to **a.** list the ordered pairs of the relation, **b.** find the domain of the relation, and **c.** find the range of the relation.

5.

Name	Birthday
Rebecca	January 18
Jennifer	February 15
John	April 1
Hector	April 7
Luis	June 23
Ebony	July 30
Raphael	August 19
Meredith	November 6
Karen	
Joseph	

6.

Name	Birthday
Amy	January 5
Carol	January 7
Devon	February 24
Harrison	March 1
Jackson	April 7
Labron	May 30
Mason	July 20
Natalie	August 1
Paul	November 13
Sylvester	November 26

7. For a woman of height $5'4''$ the mapping below shows the corresponding Body Mass Index (BMI). The body mass index is a measurement of body fat based on height and weight. A BMI of $18.5–24.9$ is considered healthy.

8. For a man of height $5'11"$ the mapping below shows the corresponding Body Mass Index (BMI).

The body mass index is a measurement of body fat based on height and weight. A BMI of $18.5–24.9$ is considered healthy.
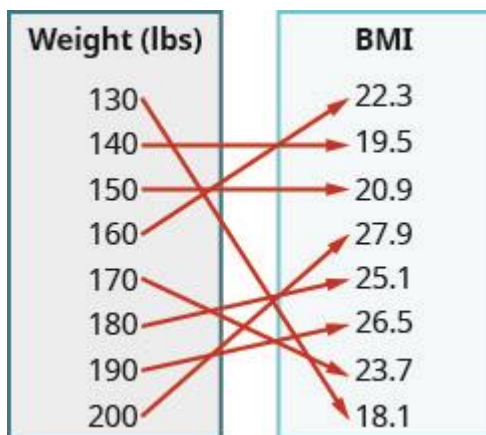
[20]

Question Seven

Prove the following propositions.

a) $n < 2^2 \ \forall \ n \geq 1$ [4]

b) $2^n < n! \ \forall \ n \geq 4$ [4]

c) $3^n < n! \ \forall \ n \geq 7$ [4]

d) $2^n > n^n \ \forall \ n \geq 5$ [4]

e) $n! < n^n \ \forall \ n \geq 2$ [4]

Question Eight

 Find the Greatest Common Divisor (GCD) of each pair using the Euclidean algorithm.
a) 275 and 115   [5]
b) 999 and 123   [5]
c) 456 and 144   [5]
d) 725 and 1000   [5]

END OF EXAMINATION