AFRICA UNIVERSITY
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"Investing in Africa's future"

COLLEGE OF ENGINEERING AND APPLIED SCIENCES (CEAS)

NCSC103: CALCULUS

END OF FIRST SEMESTER EXAMINATIONS

NOVEMBER 2023

LECTURER: DR. WESTON D GOVERE

DURATION: 3 HOURS

INSTRUCTIONS

1. *Answer **ALL** Questions from Section A and any **THREE** Questions from Section B.*
2. *All questions carry marks as indicated.*

REQUIREMENTS

- *List of Formulae and non-programmable electronic scientific calculator.*

SECTION A **(40 MARKS)**

A1

a) Prove that $\sqrt{2}$ is irrational.

b) Find a rational number whose decimal expansion is 1.63636363……. **[5, 5 marks]**

A2 The sum of terms in a geometric series is

$$\sum_{i=0}^{n} r^i = \frac{r^{n+1}-1}{r-1}, \text{if } r \neq 0, r \neq 1, n \in \mathbb{N}$$

Prove the above by the Principle of Mathematical Induction. **[8 marks]**

A3 Solve the inequality

$$|5 - \frac{2}{x}| < 1$$

[6 marks]

A4 Find the domain of the following function given by

$$f(x) = \frac{\sqrt{(3x-5)(x+4)}}{x^3 - 16x}$$

[6 marks]

A5 Evaluate the following limits

(a)

$$\lim_{x \to \frac{\pi}{4}} \frac{\sin x}{\cos x}$$

(b)

$$\lim_{x \to 0} \frac{1 - \cos 2x}{\sin 3x}$$

[4, 6 marks]

SECTION B (60 MARKS)

B6 (a) State the definition of convergence of a sequence $\{u_n\}$.

(b) Use the definition of a limit to show that

$$\lim_{n \to \infty} \frac{2n-1}{3n+2} = \frac{2}{3}$$

(c) Evaluate the following limits

(i)
$$\lim_{n\to\infty} \frac{\cos n}{n}$$

(ii)
$$\lim_{n\to\infty} \left(\sqrt{n+1} - \sqrt{n}\right)$$

(d) Show that $\lim_{n\to\infty}\left(1-\frac{1}{n}\right)^n = \frac{1}{e}$

[2, 6, 4, 4, 4 marks]

B7

a) Sketch the graph of $\sin x$

b) If $f(x) = \frac{1}{x}$, show that $f(a) - f(b) = f\left(\frac{ab}{b-a}\right)$.

c) Show that if $ad - bc \neq 0$, then the function $f(x) = \frac{ax+b}{cx+d}$ is one-to-one and find its inverse, stating the domains of both the function and its inverse.

d) State the definition of a limit of a function at a point $x = x_0$.

e) Use the definition in (d) to prove that $\lim_{x\to 1}(x^3 - 6) = -5$

[3, 5, 6, 2, 4, marks]

B8

a) Show that $f(x) = 3x - 5$ is onto.

b) If $f(x) = \sin(x + 3x^2)$ and $g(x) = \cos(x^2 - x)$. Calculate $f \circ g$ and $g \circ f$.

c) State the definition of continuity of a function $f(x)$ at a point $x = x_0$.

d) State the definition of differentiability of a function $f(x)$ at a point $x = x_0$.

e) Let

$$f(x) = \begin{cases} x\sin\left(\frac{1}{x}\right) & , \ x \neq 0 \\ 0 & , x = 0 \end{cases}$$

Use the definitions in (c) and (d) to show that $f(x)$ is continuous but not differentiable at $x = x_0$.

[4, 6, 2, 2, 6 marks]

B9 (a) Evaluate

(i) $\int \frac{1}{x} \frac{1}{(\ln x)} dx$

(ii) $\int \frac{11x + 5}{3x^2 + 2x - 5} dx$

(iii) $\int \frac{\tan^{-1} x}{1 + x^2} dx$

(b) Let $C_n = \int \cos^n ax dx$. Show that $C_n = \frac{\cos^{n-1} ax \sin ax}{n} + \frac{n-1}{n} C_{n-2}$.

Hence, show that $\int_0^{\frac{\pi}{2}} \cos^n ax dx = \frac{(n-1)(n-3)(n-5)\cdots 2}{n(n-2)(n-4)\cdots 3}$ if n is odd.

[3, 5, 3, 9 marks]

END OF EXAMINATION