AFRICA UNIVERSITY

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"Investing in Africa's future"

COLLEGE OF BUSINESS PEACE LEADERSHIP AND GOVERNANCE (CBPLG)

CSC 102: DISCRETE STRUCTURES

END OF SECOND SEMESTER EXAMINATIONS
MAY 2021
LECTURER: DR. WESTON GOVERE
TIME: HOURS

INSTRUCTIONS

Answer **ONE** question.
Each question carry **50** marks.

Show all working on your answer sheet.

The marks allocated to **each** question are shown at the end of the part question.

Question 1

a) Prove by an appropriate method that for all integers n:

$2 + 4 + 6 + \cdots + 2n = n(n + 1)$ **[3]**

b) Show that at least four of any 22 days must fall on the same day of the week. **[3]**

c) Show that the statement "Every positive integer is the sum of the squares of two integers" is false. **[3]**

d) What is wrong with this famous supposed "proof" that 1 = 2?

"Proof:" We use these steps, where a and b are two equal positive integers.

Step	**Reason**
1. $a = b$	Given
2. $a^2 = ab$	Multiply both sides of (1) by a
3. $a^2 - b^2 = ab - b^2$	Subtract b^2 from both sides of (2)
4. $(a - b)(a + b) = b(a - b)$	Factor both sides of (3)
5. $a + b = b$	Divide both sides of (4) by $a - b$
6. $2b = b$	Replace a by b in (5) because $a = b$ and simplify
7. $2 = 1$	Divide both sides of (6) by b

[3]

e) For each of the following prove using both the direct proof and contrapositive proof:

i) If n is an even integer then $3n + 5$ is odd. **[3]**

ii) Let n be an integer. If $n^2 + 5$ is odd then n is even. **[3]**

f) Show that these statements about the integer n are equivalent:

p_1: n is even.

p_2: $n - 1$ is odd.

p_3: n^2 is even. **[3]**

g) What is the probability that Abby, Barry, and Sylvia win the first, second, and third prizes, respectively, in a drawing if 200 people enter a contest and

a. no one can win more than one prize. **[2]**

b. winning more than one prize is allowed. **[3]**

h) The following table shows 1000 nursing school applicants classified according to scores made on a collage entrance examination and the quality of the high school from which they graduated, as rated by a group of educators.

	Quality of High School			
	Poor	Average	Superior	**Total**
Score	(P)	(A)	(S)	
Low (L)	105	60	55	220
Medium (M)	70	175	145	390
High (H)	25	65	300	390
Total	**200**	**300**	**500**	**1000**

For (a) – (e) calculate the probability that an applicant picked at random from this group:

i) Made a low score on the examination.
ii) Graduated from a superior high school.
iii) Made a low score on the examination and graduated from a superior high school.
iv) Made a low score on the examination given he/she graduated from a superior high school. Interpret the difference between (a) and (d).
v) Made a high score or graduated from a superior high school.

[2, 2, 2, 4, 2]

i) Show that if X and Y are independent random variables on a sample space S, then $E(XY) = E(X)E(Y)$. **[4]**

j) If X is the r.v the random number on a number a biased die. The probability mass function (p.m.f) of X is as shown

X	1	2	3	4	5	6
$P(X = x)$	1/6	1/6	1/5	a	1/5	1/6

Find

i) the value of a **[2]**

ii) E(X) **[2]**

iii) E(X^2) **[2]**

iv) Var(X) **[2]**

Question 2

a) Show that $\left(p\wedge\left(\neg(\neg p\vee q)\right)\right)\vee(p\wedge q)\equiv p$ **[7]**

b) Using Boolean algebra simplify the statement $\neg(r\to s)\to(\neg r)$ **[5]**

c) Compute the inverse, converse and contrapositive of the given statement "*If he works hard then he will pass the examination.*" **[3]**

d) Let *p* and *q* be the propositions

p :You drive over 65 miles per hour.
q :You get a speeding ticket.

Write these propositions using *p* and *q* and logical connectives (including negations).

i. Driving over 65 miles per hour is sufficient for getting a speeding ticket.
ii. You get a speeding ticket, but you do not drive over 65 miles per hour.
iii. Whenever you get a speeding ticket, you are driving over 65 miles per hour.

[3]

e) Explain, without using a truth table, why $(p\vee q\vee r)\wedge(\neg p\vee\neg q\vee\neg r)$ is true when at least one of *p*, *q*, and *r* is true and at least one is false, but is false when all three variables have the same truth value. **[3]**

f) Write each of these statements in the form "if *p*, then *q*" in English.
i. It is necessary to walk 8 miles to get to the top of Long's Peak.
ii. To get tenure as a professor, it is sufficient to be world famous.
iii. Your guarantee is good only if you bought your CD player less than 90 days ago.

[3]

g)

i) Translate the following statement into symbolic form:
"If James does not quit his job then Mary will not get any money and James' family will be happy" **[2]**

ii) Let P = I cheat; Q = I will get caught;
R = I will write an examination and S = I will fail.
Translate $(R \wedge P) \rightarrow (Q \rightarrow S)$ into simple English **[2]**

h) Show that $p \oplus q$ is equivalent to $(p \wedge \neg q) \vee (\neg p \wedge q)$. **[4]**

i) Propositions p and q are defined on the universal set $U = \{quadrilaterals\}$. Let p be the proposition "x is a square" and q be the proposition "x is a rectangle". If the implication is given as $q \rightarrow p$, then state the converse, inverse and contrapositive in words. **[3]**

j)

i. Prove that if $A \subset B$ then $A \cap B = A$. **[4]**

ii. Prove that there is only one empty set. **[4]**

k) There are 30 students in a class. Among them, 8 students are learning both English and French. A total of 18 students are learning English. If every student is learning at least one language, how many students are learning French in total? **[3]**

l) Suppose that $A = \{a, b, c, d\}$ and $B = \{1, 2\}$, write down the following sets:

i) $A \times B$ **[2]**

ii) $P(B)$ **[2]**

(I) Among a group of students, 50 played cricket, 50 played hockey and 40 played volley ball. 15 played both cricket and hockey, 20 played both hockey and volley ball, 15 played cricket and volley ball and 10 played all three. If every student played at least one game, find the number of students and how many played only cricket, only hockey and only volley ball? **[3]**

Question 3

b) Let D be the divides relation on **Z** defined as, $\forall\ m,n \in Z,\ mDn \Leftrightarrow m/n$. Determine whether D is reflexive, symmetric or transitive. Justify your answer. **[4]**

c) Determine whether the relation R on the set of all people is reflexive, symmetric, antisymmetric, and/or transitive, where $(a, b) \in R$ if and only if

i) a and b were born on the same day. **[2]**

ii) a has the same first name as b. **[2]**

iii) a and b have a common grandparent. **[2]**

d) Consider these relations on the set of integers:

i) $R_1 = \{(a, b) \mid a \leq b\}$,

ii) $R_2 = \{(a, b) \mid a > b\}$,

iii) $R_3 = \{(a, b) \mid a = b$ or $a = -b\}$,

iv) $R = \{(a, b) \mid a = b\}$,

v) $R = \{(a, b) \mid a = b + 1\}$,

vi) $R = \{(a, b) \mid a + b \leq 3\}$.

Which of these relations contain each of the pairs $(1, 1), (1, 2), (2, 1), (1, -1)$, and $(2, 2)$? **[7]**

e) Let $R_1 = (A, A, E_1)$ and $R_2 = (A, A, E_2)$ be two relations within the set ,$A = \{2,3,4,6\}$, where $E_1 = \{(x, y): x \text{ is a multiple of } y\}$ and $E_2 = \{(x, y): x \text{ and } y \text{ have no common factors }\}$

i) Write E_1 and E_2 as two sets of ordered pairs. **[2]**

ii) Determine the domain and range of R_1. **[2]**

iii) Display the diagraph of R_1. **[3]**

iv) Determine the relation matrix of R_2. **[2]**

f) Let R be the relation with ordered pairs (Abdul, 22), (Brenda, 24), (Carla, 21), (Desire, 22), (Eddie, 24), and (Felicia, 22). Here each pair consists of a graduate student and this student's age. Specify a function determined by this relation. **[3]**

g) Show that

i) $f(x) = 5x + 2$ is surjective for all $x \in \mathbb{R}$. **[3]**

ii) the function $f: \mathbb{R} \rightarrow \mathbb{R}$ defined by $f(x) = \frac{(x+1)^3}{(x-1)^3}$ is bijective.

[4]

h)

i) Data stored on a computer disk or transmitted over a data network are usually represented as a string of bytes. Each byte is made up of 8 bits. How many bytes are required to encode 100 bits of data? **[2]**

ii) In asynchronous transfer mode (ATM) (a communications protocol used on backbone networks), data are organized into cells of 53 bytes. How many ATM cells can be transmitted in 1 minute over a connection that transmits data at the rate of 500 kilobits per second? **[3]**

i) Given $f(x) = \frac{2x+1}{2-3x}$ where $\in \mathbb{R}$; $x \neq k$.

iii) State the value of k **[1]**

iv) find $f^{-1}(x)$ and clearly state the domain. **[3]**

j) If $f(x) = 5x - 5$: $x \in \mathbb{R}$ and $g(x) = \frac{1}{2x+1}$: $x \in \mathbb{R}$; $x \neq \frac{1}{2}$, show that $f\ o\ g \neq g\ o\ f$ **[2]**

k) Prove that if $f: A \rightarrow B$, $g: B \rightarrow C$ and $gof: A \rightarrow C$ is onto, then g is onto C. **[3]**

END OF EXAMINATION