COLLEGE OF BUSINESS, PEACE, LEADERSHIP AND GOVERNANCE

NCSC102: ALBEBRA

END OF SECOND SEMESTER EXAMINATIONS

MAY 2021

LECTURER: MR P TARAMBAWAMWE

DURATION: 7 HOURS

INSTRUCTIONS

You are required to answer **ONE** question.

Credit will be awarded for logical, systematic and neat presentations

Question 1

a.

i. Given B = $\begin{pmatrix} 2 & X-2 & 1 \\ 1 & 2 & -1 \\ 3 & 4-2x & -2 \end{pmatrix}$ Find X if det(B) = -6 [3]

ii. Find the inverse of matrix A if A = $\begin{pmatrix} 1 & 2 & 3 & 4 \\ -1 & 1 & 3 & -5 \\ 0 & 3 & 1 & -2 \\ 2 & 0 & 0 & -1 \end{pmatrix}$

[5]

b i.

Let A = $\begin{pmatrix} 1 & 0 & -3 & 2 \\ 0 & 3 & 1 & -5 \\ 2 & 4 & 0 & 3 \\ -3 & 1 & -1 & 2 \end{pmatrix}$ if B = $\begin{pmatrix} 1 & 2 \\ 3 & -1 \\ 0 & -2 \\ 4 & 1 \end{pmatrix}$ and C= $\begin{pmatrix} 3 & -2 & 0 & 5 \\ 1 & 0 & -3 & 4 \end{pmatrix}$

Find ABC [3]

ii. Find all values of a for which the system below has a nontrivial solution [5]

$1x - 1y + 2z = 0$

$-1x + ax + 2z = 0$

$3x - 3y + az = 0$

iii. Solve the system of linear equations below using the Gauss-Jordan method [8]

$$\begin{cases} 2w - x + 5y + z = -3 \\ 3w + 2x + 2y - 6z = -32 \\ w + 3x + 3y - z = -47 \\ 5w - 2x - 3y + 3z = 49 \end{cases}$$

iv. The prices of three commodities P,Q and R are p, q and r per units respectively. Anita purchases 8 units of Q and sells 4 units of P and 10 units of R . Abisha purchases 4 units of R and sells 6 units of P and 2 units of Q. Anesu purchases 2 units of P and sells 6 units of Q and 2 units of R . In the process, Anita, Abisha and Anesu earn $30,000, $ 2,000 and $8,000 respectively. Find the prices per unit of P,Q and R . (Use Gauss-Jordan method to solve the problem.) [8]

c

i. . Let $p(x) = x^2 + 2x - 3$, $q(x) = 2x^2 - 3x + 4$, and $r(x) = ax^2 - 1$. Find the value of a for which the set {p, q, r} is linearly dependent. [4]

ii. Show that in the vector space R the vectors x = (1, 2, −1), y = (3, 1, 1), and z = (5, −5, 7) are linearly dependent. [4]

iii.Show that in the vector space R the vectors x = (1, 2, −1), y = (3, 1, 1), and z = (5, −5, 7) are linearly dependent. [4]

iv. iLet w = (1, 1, 0, 0), x = (1, 0, 1, 0), y = (0, 0, 1, 1), and z = (0, 1, 0, 1)

find scalars α, γ, and δ such that $\alpha w + x + \gamma y + \delta z = 0$

write z as a linear combination of w, x, and y [6]

Question 2

a i.

Find the values of k for which the system of equations

$$x + ky = 1$$
$$kx + y = 1$$

has (1) no solution. Answer: . (2) exactly one solution. 3) infinitely many solutions [5]

ii. Solve the following system of linear equations [8]

$x_1 + 3x_2 + 2x_3 + 5x_4 = 11$

$-x_1 + 2x_2 - 2x_3 + 5x_4 = -6$

$2x_1 + 6x_2 + 4x_3 + 7x_4 = 19$

$5x_2 + 2x_3 + 6x_4 = 5$

iii. Eva had a bake sale to earn extra money. On the first day, she earned $12.50 selling 10 cookies and 4 brownies. On the second day, she earned $15.50 selling 6 brownies and 8 pieces of pie. On the third day, she earned $12.00 selling 16 cookies. If Eva sold 12 cookies and 2 pieces of pie the next day, how much did she make? [10]

iv. John inherited $25,000 and invested part of it in a money market account, part in municipal bonds, and part in a mutual fund. After one year, he received a total of $1,620 in simple interest from the three investments. The money market paid 6% annually, the bonds paid 7% annually, and the mutually fund paid 8% annually. There was $6,000 more invested in the bonds than the mutual funds. Find the amount John invested in each category [10]

b.

i. Show that in the space R^3 the vectors x = (1, 1, 0), y = (0, 1, 2), and z = (3, 1, −4) are linearly dependent [5]

ii. iLet w = (1, 1, 0, 0), x = (1, 0, 1, 0), y = (0, 0, 1, 1), and z = (0, 1, 0, 1) find scalars α, γ, and δ such that $\alpha w + x + \gamma y + \delta z = 0$

write z as a linear combination of w, x, and y [6]

iii. Let a, b, and c be distinct real numbers. Use the definition of linear independence to give a careful proof that the vectors (1, 1, 1), (a, b, c), and (a^2, b^2, c^2) form a linearly independent subset of R^3. [6]

Question 3

a. Solve the following system of linear equations using the Gauss Jordan method [8]

$$\begin{cases} w-5x+2y-z=-18 \\ 3w+x-3y+2z=17 \\ 4w-2x+y-z=-1 \\ -2w+3x-y+4z=11 \end{cases}$$

b. Find the additive inverse, in the vector space, of the vector [6]

i. In P^3. the vector $-3-2x+x^2$

ii. In the space 2x2

$$\begin{bmatrix} 1 & -1 \\ 0 & 3 \end{bmatrix}$$

iii. $\ln(ae^{x} + be^{-x})$ in the space of the real variable x under the natural logarithms

c. Name the zero vector for each of these vector spaces. [4]

i. The space of degree three polynomials under the natural operations

ii. The space of 2x4 matrices

d.

i. Let V be a vector space. (a) Define what it means for a set $\{u1, \dots, un\} \subset V$ to be linearly dependent [2]

ii. Define what it means for $u \in V$ to be in the span of a set $\{v1, \dots, vn\}$. [3]

iii. Prove that the vectors (1, 1, 0), (1, 2, 3), and (2, −1, 5) form a basis for R^3 [3]

e. Consider S = $[2, -3, 4, -1]^T, [-6, 9, -12, 3]^T, [3, 1, -2, 2]^T, [2, 8, -12, 3]^T, [7, 6, -10, 4]^T$

i. Is S linearly independent? [6]

ii. Construct a basis for span(S). [6]

iii. Show that in the space R^3 the vectors x = (1, 1, 0), y = (0, 1, 2), and z = (3, 1, −4) are linearly dependent [6]

iv. In the space C[0, 1] define the vectors f, g, and h by $f(x) = x$ $g(x) = e^{x}$ $h(x) = e^{-x}$ for $0 \le x \le 1$. Use the definition of linear independence to show that the functions f, g, and h are linearly independent. [6]

END OF EXAMINATION