"Investing in Africa's future"

COLLEGE OF BUSINESS PEACE LEADERSHIP AND GOVERNANCE (CBPLG)

NCSC 117**: THEORY OF COMPUTING**

END OF SECOND SEMESTER EXAMINATIONS

MAY 2021

LECTURER: DR. WESTON GOVERE

TIME: 7 HOURS

INSTRUCTIONS

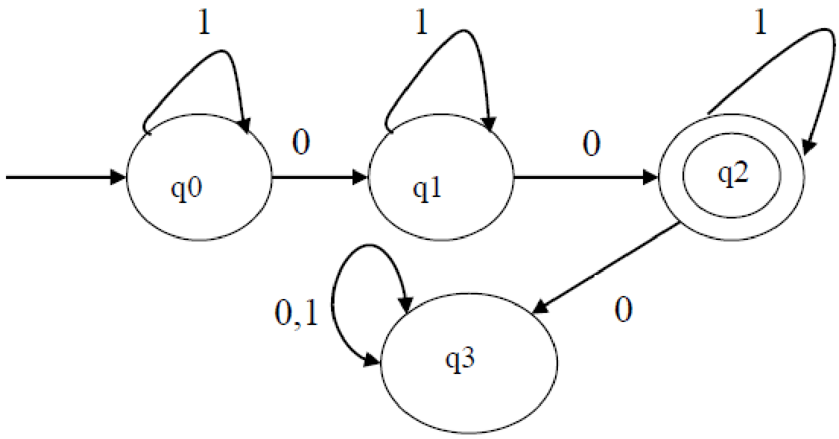
Answer **ONE** question.
Each question carries **50** marks.

Show all working on your answer sheet.

The marks allocated to **each** question are shown at the end of the part question.

Question 1

(a) Let M be the Deterministic Finite Automata (DFA) shown below



i. Provide a formal description of M.

ii. In plain English, describe the language described by this DFA. In order to get full credit, you need to provide reasonably succinct description that ignores irrelevant considerations.

iii. Is the language described in part ii above a context-free language? Briefly justify your answer.

[6, 4, 4]

(b)

i. Draw the Nondeterministic Finite Automata (NFA) that recognizes the language where w contains the substring 0101. Do this using 5 states and assuming a binary alphabet.

ii. Prove or disprove the following statements. Assume that $\Sigma = \{0,1\}$

a. 0^n1^{2n} is regular, $n \geq 1$

b. 0^n1^{2n} is context-free, $n \geq 1$

[7, 5, 4]

(c) Each of these "proofs" claims to prove a conjecture that is false. For each proof, identify the first step that is wrong, and briefly and clearly explain why.

i. **False Theorem:** The language

$L = \{w \mid w \in \{0; 1\}^*$ and the number of 1s in w is odd$\}$ is non-regular.

Claimed Proof. We use proof-by-contradiction to show L is not regular.

1. Assume L is regular.
2. Since L is regular, the pumping lemma should be satisfied.
3. Choose s = 1, which satisfies the requirements for the pumping lemma since it is in L (it has an odd number of 1s).
4. The only way to satisfy the pumping lemma requirements for dividing s = xyz is to make $x = \epsilon$, y = 1, and $z = \epsilon$ since the pumping lemma requires $|y| > 0$.
5. But, this division leads to a contradiction of the pumping lemma, since for i = 2, $xy^iz = 11$ which is not in L (it has an even number of 1s).
6. Since the pumping lemma is not satisfied, the assumption must be invalid.
7. Thus, L is not regular.

ii. **False Theorem:** All regular languages include the empty string.

Proof by induction. We induce on the size of the language.

1. Basis: A language of size 1 includes the empty string: the language $\{ \epsilon \}$ is a regular language and it includes the empty string.
2. Induction: We assume that all regular languages of size n include the empty string, and show that all regular languages of size n + 1 include the empty string.
2a. Suppose L is a language of size n + 1.
2b. For some language M of size n, $L = M \cup w$ where *w* is some string not in M.
2c. By the induction hypothesis, M includes the empty string, so $L = M \cup w$ also
includes the empty string.

iii. **False Theorem:** If A and B are context-free languages, then $A \cap B$ is a context free language.

Proof by construction.

1. Since A and B are context-free languages, there exist deterministic pushdown automata $P_A = (Q_A, \Sigma, \Gamma, \delta_A, q_0A, F_A)$ and $P_B = (Q_B, \Sigma, \Gamma, \delta_B, q_0B, F_B)$ that recognize A and B respectively.

2. We can construct the deterministic pushdown automaton $P_{AB} = (Q_{AB}, \Sigma, \Gamma_{AB}, \delta_{AB}, q_{0AB}, F_{AB})$ that recognizes the intersection of A and B by simulating both P_A and P_B simultaneously:

2a. $Q_{AB} = Q_A \times Q_B$
2b. $\Gamma_{AB} = \Gamma_A \times \Gamma_B$
2c. $\delta_{AB}((q_a, q_b), \sigma, (\gamma_a, \gamma_b)) = ((r_a, r_b), (y_a, y_b))$
where $\delta_A(q_a, \sigma, \gamma_a) = (r_a, y_a)$ and $\delta_B(q_b, \sigma, \gamma_b) = (r_b, y_b)$
2d. $q_{0AB} = (q_0A, q_0B)$
2e. $F_{AB} = \{(q_a, q_b) | q_a \in F_A \wedge q_b \in F_B\}$

3. Since P_{AB} accepts only if both P_A and P_B would accept, the language recognized by P_{AB} is the intersection of the languages recognized by P_A and P_B.

[7, 5, 8]

Question 2

(a) Assume an alphabet ∑ that is {0, 1}

i. Draw the *simplest* possible DFA(in terms of number of states and arcs) that describes the language of all strings that end in “00”.
ii. Provide the regular expression that describes the language in part i.

[7, 3]

(b) For each described language, your answer should identify the simplest machine that can recognize the given language. The choices for machines (from simplest to least simple) are:

(A) Deterministic Finite Automaton with no cycles
(B) Deterministic Finite Automaton
(C) Deterministic Push-Down Automaton
(D) Nondeterministic Push-Down Automaton
(E) None of the Above

Your answer should include one of the five letters to identify the machine, and a brief justification. For these questions, you are not required to actually construct the machine or provide a proof for a full credit answer; a correct intuitive explanation is sufficient.

i. {w | w describes the position of a chess board in which White can win }

ii. The language of all strings generated by the grammar: $S \rightarrow 0S \mid 1S \mid \varepsilon$.

iii. $\{0^i 1^{2i} 0^{3i} \mid i \geq 0\}$

iv. The language of all strings in {a, b}* that do not include any substring of the form $a^i b^i$ where $i \geq 1$.

[5, 5, 5, 5]

(c) Each of these "proofs" claims to prove a conjecture that is false. For each proof, identify the *first step* that is wrong, and briefly and clearly explain why. The explanation is more important than the step you identify.

i. **False Conjecture:** The intersection of two context free languages is context free.

Claimed Proof.

1. We define the intersection of two languages

$$A \cap B = \{w | w \in A \wedge w \in B\}.$$

2. The language $X = A \cap B$ is a subset of the language A: $X \subseteq A$. This is true since every string in X must also be in A.

3. Since X is a subset of A, and A is context free, X is context free.

ii. **False Conjecture:** All whole numbers are even.

Claimed Proof. We use induction on the numbers to prove the conjecture.

1. A number n is *even* if $2m = n$ for some integer m.
2. *Basis:* 0 is even. Select $m = 0$, then $2m = 0$.
3. *Induction:* Assume the conjecture holds for all $i < n$. We show that it holds for n.
4. $n = k + 2$ for some integer $k < n$.
5. By the induction hypothesis, $k = 2m$ for some integer m.
6. $n = k + 2 = (2m) + 2 = 2(m + 1)$. Thus, n is even.

iii. **False Conjecture:** The language TRIPLES is not regular.

$$TRIPLES = \left\{1^{3n} | n \geq 0\right\}$$

Claimed Proof. We use the pumping lemma for regular languages to obtain a contradiction.

1. Assume $TRIPLES$ is regular. Then, there exists some DFA M with pumping length p that recognizes $TRIPLES$.
2. Choose $s = 1^p$.
3. The pumping lemma requires that there is a way to divide s into $s = xyz$ where $|y| \geq 1$ and $|xy| \leq p$ and $xy^iz \in TRIPLES$ for all $i \geq 0$.
4. Since s is all 1s, we know y can contain only 1s.
5. Choose $y = 1$.
6. Choose $i = 2$.
7. Since xy^2z now has $p + 1$ ones, it is not in the language $TRIPLES$.
8. Thus, we have a contradiction of the pumping lemma and $TRIPLES$ must not be regular.

[5, 7, 8]

END OF EXAMINATION PAPER