AFRICA UNIVERSITY
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COLLEGE OF BUSINESS, PEACE, LEADERSHIP AND GOVERNANCE (CBPLG)

CALCULUS –NCSC 103

SUPPLEMENTARY EXAMINATIONS

JANUARY 2020

LECTURER: Mr.Timothy Makambwa

DURATION: 3 HOURS

INSTRUCTIONS

Answer **ALL** the questions in this Examination Paper. Total possible mark is **100.**

Start **each** question on a new page on your answer sheet.

The marks allocated to **each** question are shown at the end of the section.

Question One

Compute the following limits. Each limit is worth 5 marks. **Note** remember to simplify your answers.

a) $\lim_{x \to -2} \frac{x^2+3x+2}{x+2}$

b) $\lim_{n \to \infty} \frac{1+2+3+....n}{n^2}$

c) $\lim_{x \to} \frac{1}{x-1} - \frac{2}{x^2-1}$

d) $\lim_{x \to 0^+} \frac{\sin(6x)}{x}$

e) $\lim_{h \to 0} \frac{(1+h)^2-1}{h}$

[25]

Question Two

Differentiate.

a) $f(x) = e^x + x^3 - 5\ln x$

b) $g(q) = 8q^5 + 7q^4 - \dfrac{4}{7q^3}$

c) $y = \sqrt[5]{x} - \dfrac{2}{x} + 4e^x$

d. Find $\dfrac{d^3y}{dx^3}$ for $y = 2x^5 + 3x^3 + 9x - 5e^x$

[15]

Question Three

Integrate the following by making the substitution given

a) $\int e^{-x}dx$, $u=-x$ [3]

b) $\int x^3(2+x^4)^5dx$, $u= 2+x^4$ [4]

c) $\int x^2\sqrt{(x^3+1)}\, dx$, $u= x^3+1$ [4]

d) $\int \cos^3\theta \sin\theta\, d\theta$, $u= \cos\theta$ [4]

e) $\int \frac{\sec^2(1/x)}{x^2} dx$, $u= 1/x$ [5]

Question Four

a) Find the equation of the tangent and normal to the demand curve $y = 10-3x^2$ at (1, 7).

b) Find the slope of the tangent line at the point (0, 5) of the curve $y = ⅓ (x^2 + 10x – 15)$. At what point of the curve the slope of the tangent line is 5 /8 ?

c) Determine the coefficients a and b so that the curve $y = ax^2 -6x + b$ may pass through the point (0, 2) and have its tangent parallel to the x-axis at $x = 1.5$.

d) Determine the values of l and m so that the curve, $y = lx^2 + 3x + m$ may pass through the point (0, 1) and have its tangent parallel to the x-axis at x = 0.75.

[5+5+5+5]

Question Five

a) Investigate the maxima and minima of the function $2x^3 + 3x^2 - 36x + 10$.
b) Find the absolute (global) maximum and minimum values of the function $f(x) = 3x^5 - 25x^3 + 60x + 1$ in the interval [-2, 1]
c) Find the points of inflection of the curve $y = 2x^4 - 4x^3 + 3$.

[6+8+6]

END OF PAPER

Mathematical Formulae

Integration Techniques

Many integration formulas can be derived directly from their corresponding derivative formulas, while other integration problems require more work. Some that require more work are substitution and change of variables, integration by parts, trigonometric integrals, and trigonometric substitutions.

Basic formulas

Most of the following basic formulas directly follow the differentiation rules.

1. $\int kf(x)\,dx = k\int f(x)\,dx$
2. $\int [f(x) \pm g(x)]\,dx = \int f(x)\,dx \pm \int g(x)\,dx$
3. $\int k\,dx = kx + C$
4. $\int x^n\,dx = \frac{x^{n+1}}{n+1} + C, n \neq -1$
5. $\int \sin x\,dx = -\cos x + C$
6. $\int \cos x\,dx = \sin x + C$
7. $\int \sec^2 x\,dx = \tan x + C$
8. $\int \csc^2 x\,dx = -\cot x + C$
9. $\int \sec x \tan x\,dx = \sec x + C$
10. $\int \csc x \cot x\,dx = -\csc x + C$
11. $\int e^x\,dx = e^x + C$
12. $\int a^x\,dx = \frac{a^x}{\ln a} + C, a > 0, a \neq 1$
13. $\int \frac{dx}{x} = \ln|x| + C$
14. $\int \tan x\,dx = -\ln|\cos x| + C$
15. $\int \cot x\,dx = \ln|\sin x| + C$
16. $\int \sec x\,dx = \ln|\sec x + \tan x| + C$
17. $\int \csc x\,dx = -\ln|\csc x + \cot x| + C$
18. $\int \frac{dx}{\sqrt{a^2 - x^2}} = \arcsin\frac{x}{a} + C$
19. $\int \frac{dx}{a^2 + x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$

20. $\int \frac{dx}{x\sqrt{x^2 - a^2}} = \frac{1}{a}\operatorname{arcsec}\frac{x}{a} + C$

Applications of Integration

$\therefore$ Producer Surplus = p_0q_0 - $\int gx\,dx$ with limits from 0 to q_0

Consumer Surplus = $\int gx\,dx - p_0q_0$ with limits from 0 to q_0

Total Cost = $\int$ MCdx

Total Revenue = $\int$MRdx

Differentiation

The chain rule

If *y* is a function of *u*, which is itself a function of *x*, then

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$$

differentiate the outer function and multiply by the derivative of the inner function

The product rule

If $y = uv$ then = $\frac{dy}{dx} = u.\frac{dv}{dx}$ x $v.\frac{du}{dx}$

This rule tells you how to differentiate the product of two functions: **multiply each function by the derivative of the other and add**

The quotient rule

If $y = \frac{u}{v}$ then $\frac{dy}{dx} = \frac{v.du/dx - u.dv/dx}{v^2}$

This rule tells you how to differentiate the quotient of two functions: **bottom times derivative of top, minus top times derivative of bottom, all over bottom squared**