AFRICA UNIVERSITY
(A United Methodist-Related Institution)

"Investing in Africa's Future"

COLLEGE OF BUSINESS, PEACE, LEADERSHIP AND GOVERNANCE

Quantitative Analysis I (NMMS202)

Supplementary Examination – January 2020

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3 Hours

INSTRUCTIONS

Answer **All** questions.

Start **each** question on a new page in your answer booklet.

The marks allocated are shown at the end of **each** question.

Show all your workings where appropriate.

Credit will be given for logical, systematic and neat presentations.

1. Define the following Quantitative Analysis I terms: **[12 marks]**
   (a) Open-ended questions.
   (c) Snowball sampling.
   (d) Panel survey.
   (e) Random sampling.

2. Do government employees take longer breaks than private sector workers? A management consultant is interested to investigate the question. To examine the issue, the consultant took a random sample of ten government employees (GE) and another random sample of six private sector workers (PSW) and measured the amount of time in minutes they spent in coffee breaks during the day. The results are shown in the following table:

GE	23	18	34	31	28	33	25	27	32	21
PSW	25	19	18	22	28	25	21	21	20	16

   Assuming both populations are approximately normal with equal variances, do the data provide sufficient evidence at 5% level of significance to support the claim? Find the p-value.
   [9 marks]

3. Twenty percent of the population are thought to be carriers of a certain disease, although they themselves may show no symptoms. If this is true, evaluate the following probabilities for a sample of five people selected at random from the population:
   i) that none are carriers. **[2 marks]**
   ii) that all five are carriers. **[2 marks]**
   iii) that fewer than two are carriers. **[3 marks]**
   iv) State the variation and expected values of the distribution. **[3 marks]**

4. The followings data shows the information of serving time (in minutes) for 40 customers in a post office:

2.0	4.5	2.5	2.9	4.2	2.9	3.5	2.8
3.2	2.9	4.0	3.0	3.8	2.5	2.3	3.5
2.1	3.1	3.6	4.3	4.7	2.6	4.1	3.1
4.6	2.8	5.1	2.7	2.6	4.4	3.5	3.0
2.7	3.9	2.9	2.9	2.5	3.7	3.3	2.4

   a. Construct a frequency distribution table. **[5 Marks]**
   b. Using the frequency table you constructed in (a):
      i) Construct a histogram and explain its purpose. **[6 Marks]**
      ii) Calculate the mode and median of the data. **[8 Marks]**
      iii) Find the mean serving time. **[3 Marks]**
      iv) Determine the skewness of the data and interpret. **[3 Marks]**
      v) Find the lower and upper quartiles of the data and determine the inter-quartile range. **[6 Marks]**

5. A survey classified 200 students by gender and by their opinion ("For" or "Against") on a certain issue. The number falling into the different categories are shown in the following table. A student is randomly selected from the group.

	Opinion	
Gender	For	Total
Female	30	70
Male	50	130
Total	80	200

Find the probability that a student selected is
a) a female and is against the issue **[3 Marks]**
b) a male or is for the issue **[2 Marks]**
c) for the issue given that the student is male **[3 Marks]**

6. Suppose 15% of a population has flu and the researcher takes a random sample of 6 people from the population. If X is the number of people in the sample with flu, then the probability density function for X is

x	0	1	2	3	4	5	6
$P(x)$	0.3771	0.3993	0.1762	0.0415	0.0055	0.0004	0.0000

a) Find the probability that at least 2 people in the sample have flu **[2 Marks]**
b) 15% of the population has the flu. Is it always the case that 15% of the sample has flu? Why? **[3 Marks]**
c) Find the expected value and variance of X **[5 Marks]**

7. It has been decided to compare some of the results from the Arbour Housing survey (Survey 1) with those from the Pelouse Housing survey (Survey 2). Of particular interest was the level of monthly rent, a summary of which is given below:

Arbour housing survey	Pelouse Housing survey
Sample size = 180	Sample size = 150
Sample mean = \$221.25	Sample mean = \$206. 38
Sample variance = \$89.48	Sample variance = \$69.88

Is there a significant difference between the two surveys?
a) Use the three methods of testing hypothesis to test the claim at 5 percent level of significance. Comment your results **[14 Marks]**
b) With a diagram and examples, explain the relationship between statistic and parameter **[6 Marks]**

End of paper

ADDITIONAL INFORMATION

1. Sturge's Rule:

   Number of class, $C = 1 + 3.3\log n$

   Class width, $i > \frac{range}{C}$

2. Mean of grouped data $= \frac{\sum_{i=1}^{n} fx_i}{n}$

3. Mean of ungrouped data $= \frac{\sum_{i=1}^{n} x_i}{n}$

4. Mode = $L_{mo} + \left(\frac{\Delta_1}{\Delta_1 + \Delta_2}\right)i$

5 Median = $L_{me} + \left(\frac{\frac{n}{2} - F}{f_m}\right)i$

6. Standard deviation: $s = \sqrt{\frac{\sum_{i=1}^{n} fx_i^2 - \frac{\left(\sum_{n=1}^{n} fx_i\right)^2}{n}}{n-1}}$

7. Standard Deviation of ungrouped data: $s = \sqrt{\frac{\sum_{i=1}^{n} x_i^2 - \frac{\left(\sum_{i=1}^{n} x_i\right)^2}{n}}{n-1}}$

8. Coefficient of skewness: $S_k = \frac{3(mean - median)}{s} = \frac{mean - \text{mod}e}{s}$

9. Conditional probability: $P(A \setminus B) = \frac{P(A \cap B)}{P(A)}$

10. Binomial Distribution
    - $P(X = x) = nC_x p^x q^{n-x}$

11. Poisson Distribution
    - $P(X = x) = \frac{e^{-\lambda}\lambda^x}{x!}$

12. Hypothesis testing (single mean)
    - $Z = \frac{\bar{X} - \mu}{\frac{\sigma}{\sqrt{n}}}$
    - $t = \frac{\bar{X} - \mu}{\frac{s}{\sqrt{n}}}$, $df = n - 1$

13. Hypothesis testing (single proportion)
    - $Z = \frac{p - \pi}{\sqrt{\frac{\pi(1-\pi)}{n}}}$

14. Hypothesis testing (difference of two means)

- $Z = \frac{\bar{X}_1 - \bar{X}_2}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$
- $t = \frac{\bar{X}_1 - \bar{X}_2}{\sqrt{\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}}}$, *df* = smaller $(n_1 - 1;\ n_2 - 1)$
- $t = \frac{\bar{X}_1 - \bar{X}_2}{\sqrt{\frac{S_p^2}{n_1} + \frac{S_p^2}{n_2}}}$

  Where $S_p^2 = \frac{S_1^2(n_1) + S_2^2(n_2)}{n_1 + n_2 - 2}$, *df* = $n_1 + n_2 - 2$
- $t = \frac{\bar{D} - \mu_D}{\frac{S_D}{\sqrt{n}}}$, *df* = n – 1

15. Hypothesis testing (difference of two proportions)
    - $Z = \frac{p_1 - p_2}{\sqrt{\bar{p}\bar{q}\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$
16. Confidence Interval (Single mean)
    - $\bar{X} - Z_{\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}} \leq \mu \leq \bar{X} + Z_{\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}}$
    - $\bar{X} - t_{\frac{\alpha}{2}} \frac{S}{\sqrt{n}} \leq \mu \leq \bar{X} + t_{\frac{\alpha}{2}} \frac{S}{\sqrt{n}}$
17. Confidence Interval (Difference of two means)
    - $(\bar{X}_1 - \bar{X}_2) - Z_{\frac{\alpha}{2}} \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}} \leq (\mu_1 - \mu_2) \leq (\bar{X}_1 - \bar{X}_2) + Z_{\frac{\alpha}{2}} \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$
    - $(\bar{X}_1 - \bar{X}_2) - t_{\frac{\alpha}{2}} \sqrt{\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}} \leq (\mu_1 - \mu_2) \leq (\bar{X}_1 - \bar{X}_2) + t_{\frac{\alpha}{2}} \sqrt{\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}}$

      df = smaller $(n_1 - 1;\ n_2 - 1)$
    - $(\bar{X}_1 - \bar{X}_2) - t_{\frac{\alpha}{2}} \sqrt{\frac{S_p^2}{n_1} + \frac{S_p^2}{n_2}} \leq (\mu_1 - \mu_2) \leq (\bar{X}_1 - \bar{X}_2) + t_{\frac{\alpha}{2}} \sqrt{\frac{S_p^2}{n_1} + \frac{S_p^2}{n_2}}$

      Where $S_p^2 = \frac{S_1^2(n_1) + S_2^2(n_2)}{n_1 + n_2 - 2}$, *df* = $n_1 + n_2 - 2$
18. Confidence Interval (Single proportion)
    - $p - Z_{\frac{\alpha}{2}} \sqrt{\frac{pq}{n}} \leq \pi \leq p + Z_{\frac{\alpha}{2}} \sqrt{\frac{pq}{n}}$
19. Confidence Interval (Difference of two proportions)
    - $(p_1 - p_2) - Z_{\frac{\alpha}{2}} \sqrt{\frac{p_1 q_1}{n_1} + \frac{p_2 q_2}{n_2}} \leq (\pi_1 - \mu_2) \leq (p_1 - p_2) + Z_{\frac{\alpha}{2}} \sqrt{\frac{p_1 q_1}{n_1} + \frac{p_2 q_2}{n_2}}$
20. $Z = \frac{X - \mu}{\sigma}$
21. Weighted Mean: $\bar{X}_w = \frac{\sum xw}{\sum w}$
22. $P(B_i / C) = \frac{P(C / B_i) P(B_i)}{\sum_{i=1}^{n} P(C / B_i) P(B_i)}$,

23. $y = \beta_0 + \beta_1 x + e,$

$$\beta_1 = \frac{n\sum xy - \sum x \sum y}{n\sum y^2 - (\sum y)^2}$$

$$\beta_0 = \bar{y} + \beta_1 \bar{x}$$

$$r = \frac{n\sum xy - \sum x \sum y}{\{(n\sum x^2 - (\sum x)^2)(n\sum y^2 - (\sum y)^2)\}}$$

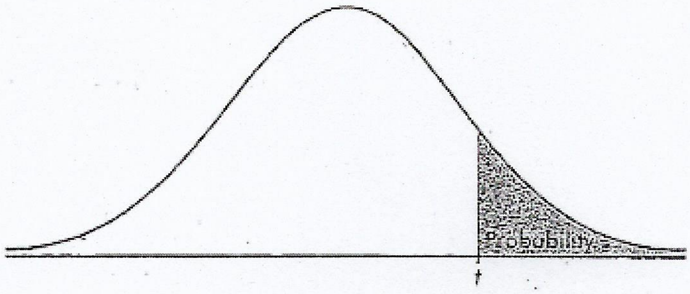


TABLE B: *t*-DISTRIBUTION CRITICAL VALUES

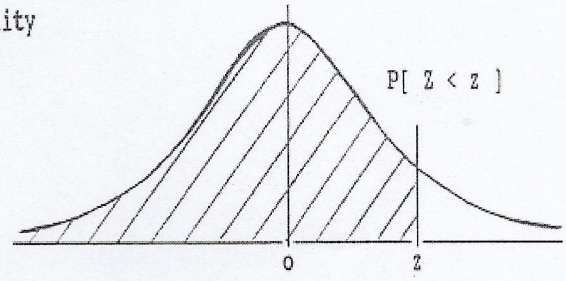
	Tail probability p											
df	.25	.20	.15	.10	.05	.025	.02	.01	.005	.0025	.001	.0005
1	1.000	1.376	1.963	3.078	6.314	12.71	15.89	31.82	63.66	127.3	318.3	636.6
2	.816	1.061	1.386	1.886	2.920	4.303	4.849	6.965	9.925	14.09	22.33	31.60
3	.765	.978	1.250	1.638	2.353	3.182	3.482	4.541	5.841	7.453	10.21	12.92
4	.741	.941	1.190	1.533	2.132	2.776	2.999	3.747	4.604	5.598	7.173	8.610
5	.727	.920	1.156	1.476	2.015	2.571	2.757	3.365	4.032	4.773	5.893	6.869
6	.718	.906	1.134	1.440	1.943	2.447	2.612	3.143	3.707	4.317	5.208	5.959
7	.711	.896	1.119	1.415	1.895	2.365	2.517	2.998	3.499	4.029	4.785	5.408
8	.706	.889	1.108	1.397	1.860	2.306	2.449	2.896	3.355	3.833	4.501	5.041
9	.703	.883	1.100	1.383	1.833	2.262	2.398	2.821	3.250	3.690	4.297	4.781
10	.700	.879	1.093	1.372	1.812	2.228	2.359	2.764	3.169	3.581	4.144	4.587
11	.697	.876	1.088	1.363	1.796	2.201	2.328	2.718	3.106	3.497	4.025	4.437
12	.695	.873	1.083	1.356	1.782	2.179	2.303	2.681	3.055	3.428	3.930	4.318
13	.694	.870	1.079	1.350	1.771	2.160	2.282	2.650	3.012	3.372	3.852	4.221
14	.692	.868	1.076	1.345	1.761	2.145	2.264	2.624	2.977	3.326	3.787	4.140
15	.691	.866	1.074	1.341	1.753	2.131	2.249	2.602	2.947	3.286	3.733	4.073
16	.690	.865	1.071	1.337	1.746	2.120	2.235	2.583	2.921	3.252	3.686	4.015
17	.689	.863	1.069	1.333	1.740	2.110	2.224	2.567	2.898	3.222	3.646	3.965
18	.688	.862	1.067	1.330	1.734	2.101	2.214	2.552	2.878	3.197	3.611	3.922
19	.688	.861	1.066	1.328	1.729	2.093	2.205	2.539	2.861	3.174	3.579	3.883
20	.687	.860	1.064	1.325	1.725	2.086	2.197	2.528	2.845	3.153	3.552	3.850
21	.686	.859	1.063	1.323	1.721	2.080	2.189	2.518	2.831	3.135	3.527	3.819
22	.686	.858	1.061	1.321	1.717	2.074	2.183	2.508	2.819	3.119	3.505	3.792
23	.685	.858	1.060	1.319	1.714	2.069	2.177	2.500	2.807	3.104	3.485	3.768
24	.685	.857	1.059	1.318	1.711	2.064	2.172	2.492	2.797	3.091	3.467	3.745
25	.684	.856	1.058	1.316	1.708	2.060	2.167	2.485	2.787	3.078	3.450	3.725
26	.684	.856	1.058	1.315	1.706	2.056	2.162	2.479	2.779	3.067	3.435	3.707
27	.684	.855	1.057	1.314	1.703	2.052	2.158	2.473	2.771	3.057	3.421	3.690
28	.683	.855	1.056	1.313	1.701	2.048	2.154	2.467	2.763	3.047	3.408	3.674
29	.683	.854	1.055	1.311	1.699	2.045	2.150	2.462	2.756	3.038	3.396	3.659
30	.683	.854	1.055	1.310	1.697	2.042	2.147	2.457	2.750	3.030	3.385	3.646
40	.681	.851	1.050	1.303	1.684	2.021	2.123	2.423	2.704	2.971	3.307	3.551
50	.679	.849	1.047	1.299	1.676	2.009	2.109	2.403	2.678	2.937	3.261	3.496
60	.679	.848	1.045	1.296	1.671	2.000	2.099	2.390	2.660	2.915	3.232	3.460
80	.678	.846	1.043	1.292	1.664	1.990	2.088	2.374	2.639	2.887	3.195	3.416
100	.677	.845	1.042	1.290	1.660	1.984	2.081	2.364	2.626	2.871	3.174	3.390
1000	.675	.842	1.037	1.282	1.646	1.962	2.056	2.330	2.581	2.813	3.098	3.300
∞	.674	.841	1.036	1.282	1.645	1.960	2.054	2.326	2.576	2.807	3.091	3.291
	50%	60%	70%	80%	90%	95%	96%	98%	99%	99.5%	99.8%	99.9%
	Confidence level C											

STANDARD STATISTICAL TABLES

1. Areas under the Normal Distribution

The table gives the cumulative probability up to the standardised normal value z
i.e.

$$P[\ Z < z\ ] = \int_{-\infty}^{z} \frac{1}{\sqrt{2\pi}} \exp(-\tfrac{1}{2}Z^2)\ dZ$$



z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5159	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7854
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8804	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9773	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9865	0.9868	0.9871	0.9874	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9924	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9980	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986

z	3.00	3.10	3.20	3.30	3.40	3.50	3.60	3.70	3.80	3.90
P	0.9986	0.9990	0.9993	0.9995	0.9997	0.9998	0.9998	0.9999	0.9999	1.0000