"Investing in Africa's Future"

COLLEGE OF ENGINEERING AND APPLIED SCIENCES

NCSC 103: CALCULUS

END OF FIRST SEMESTER EXAMINATION

NOVEMBER 2024

LECTURER: Mr. Timothy Makambwa

DURATION: 3 HOURS

INSTRUCTIONS

Answer ***All*** the questions in Section A and any three from Section B

Total possible mark is **100.**

Start **each** question on a new page on your answer sheet.

The marks allocated to **each** question are shown at the end of the section.

Section A – (Compulsory 40 Marks)

Question One

Find the following limits.

a) $\lim_{x\to\infty} \frac{3x - x^3}{2x^3 + 1}$ [3]

b) $\lim_{x\to -2} \frac{x-3}{x^2 - x - 6}$ [3]

c) $\lim_{x\to -5} (-x^2 + x)$ [2]

1. Differentiate.

a) $f(x) = e^x + x^3 - 5\ln x$ [3]

b) $g(q) = 8q^5 + 7q^4 - \frac{4}{7q^3}$ [3]

c) $y = \sqrt[5]{x} - \frac{2}{x} + 4e^x$ [3]

d. Find $\frac{d^3y}{dx^3}$ for $y = 2x^5 + 3x^3 + 9x - 5e^x$ [4]

Question Three

a) $\int (e^x - x^2 - 5)dx$ [3]

b) $\int_1^e (x^2 - \frac{1}{x})dx$ [4]

c) $\int (\frac{6}{x} + \frac{4}{x^4} - \sqrt{x})dx$ [4]

d) $\int_{-1}^{1} (t^2 - t^4)dt$ [4]

e) find $f(x)$ if $f'(x) = \sqrt{x} - 3$ and $f(4) = -1$. . [4]

Section B (60 Marks)

Answer *Any* Three questions

Question Four

a) A monopoly faces the following TR and TC schedules:

$TR = 300q - 2q^2$

$TC = 12q^3 - 44q^2 + 60q + 30$

What output should it sell to maximize profit? [10]

b) A firm faces the demand function $p = 190 - 0.6q$ and the total cost function $TC = 40 + 30q + 0.4q^2$

i. What output will maximize profit?

ii. What output will maximize total revenue?

iii. What will the output be if the firm makes a profit of £4,760? [10]

Question Five

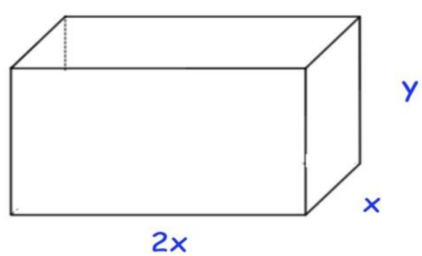
a) Find the area under the curve of the function y = $y = x^2 - 4x + 5$ from x = -1 to x =3. Sketch the graph [5]

b) Investigate the maxima and minima of the function $2x^3 + 3x^2 - 36x + 10$. [7]

c) Find the absolute (global) maximum and minimum values of the function $f(x) = 3x^5 - 25x^3 + 60x + 1$ in the interval [-2, 1] [8]

Question Six

a) An open-topped tank in the shape of a cuboid is shown below.



The surface area of the cuboid is 300cm²

i. Show that $y = \frac{50}{x} - \frac{x}{3}$ [3]

ii. Show that the volume of the tank is $V = 100x - \frac{2}{3}x^3$ [3]

iii. Use differentiation to find the value of x for which V is a maximum [4]

iv. Find the volume of the tank [2]

b) After t hours of operation, a coal mine is producing coal at a rate of $40 + 2t - 9t^2$ tons of coal per hour. Find the formula for the output of the coal mine after t hours of operation if we know that after 2 hours, 80 tons have been mined. [8]

Question Seven

a) For the marginal cost function $MC = 5 - 6x + 3x^2$, x is the output. If the cost of producing 10 items is Rs.850, find the total cost and average cost function. [5]

b) The marginal cost function is $MC = 20 - 0.04x + 0.003x^2$ where x is the number of units produced. The fixed cost of production is Rs. 7,000. Find the total cost and the average cost. [5]

c) If the marginal revenue function is $R¢(x) = 15 - 9x - 3x^2$, find the revenue function and average revenue function. [5]

d) If the marginal revenue of a commodity is given by $MR = 9 - 2x + 4x^2$, find the demand function and revenue function. [5]

Question Eight

a) The demand and supply function for a commodity are $p_d = 16 - 2x$ and $p_s = x^2 + 1$. Find the consumers' surplus and producers' surplus at the market equilibrium price. [10]

b) The demand and supply law under a pure competition are given by $p_d = 23 - x^2$ and $p_s = 2x^2 - 4$. Find the consumers' surplus and producers' surplus at the market equilibrium price. [10]

END OF EXAMINATION

Mathematical Formulae

Integration Techniques

Many integration formulas can be derived directly from their corresponding derivative formulas, while other integration problems require more work. Some that require more work are substitution and change of variables, integration by parts, trigonometric integrals, and trigonometric substitutions.

Basic formulas

Most of the following basic formulas directly follow the differentiation rules.

1. $\int kf(x)\,dx = k\int f(x)\,dx$
2. $\int [f(x) \pm g(x)]\,dx = \int f(x)\,dx \pm \int g(x)\,dx$
3. $\int k\,dx = kx + C$
4. $\int x^n\,dx = \frac{x^{n+1}}{n+1} + C, n \neq -1$
5. $\int \sin x\,dx = -\cos x + C$
6. $\int \cos x\,dx = \sin x + C$
7. $\int \sec^2 x\,dx = \tan x + C$
8. $\int \csc^2 x\,dx = -\cot x + C$
9. $\int \sec x \tan x\,dx = \sec x + C$
10. $\int \csc x \cot x\,dx = -\csc x + C$
11. $\int e^x\,dx = e^x + C$
12. $\int a^x\,dx = \frac{a^x}{\ln a} + C, a > 0, a \neq 1$
13. $\int \frac{dx}{x} = \ln|x| + C$
14. $\int \tan x\,dx = -\ln|\cos x| + C$
15. $\int \cot x\,dx = \ln|\sin x| + C$
16. $\int \sec x\,dx = \ln|\sec x + \tan x| + C$
17. $\int \csc x\,dx = -\ln|\csc x + \cot x| + C$
18. $\int \frac{dx}{\sqrt{a^2 - x^2}} = \arcsin\frac{x}{a} + C$
19. $\int \frac{dx}{a^2 + x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$

20. $\int \frac{dx}{x\sqrt{x^2 - a^2}} = \frac{1}{a} \operatorname{arcsec} \frac{x}{a} + C$

Applications of Integration

$\therefore$ Producer Surplus = $p_0q_0 - \int gx\,dx$ with limits from 0 to q_0

Consumer Surplus = $\int gx\,dx - p_0q_0$ with limits from 0 to q_0

Total Cost = $\int$ MCdx

Total Revenue = $\int$MRdx

Differentiation

The chain rule

If *y* is a function of *u*, which is itself a function of *x*, then

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$$

differentiate the outer function and multiply by the derivative of the inner function

The product rule

If $y = uv$ then = $\cdot \frac{dy}{dx} = u.\frac{dv}{dx} \quad x \quad v.\frac{du}{dx}$

This rule tells you how to differentiate the product of two functions: **multiply each function by the derivative of the other and add**

The quotient rule

If $y = \frac{u}{v}$ then $\frac{dy}{dx} = \frac{v.du/dx - u.dv/dx}{v^2}$

This rule tells you how to differentiate the quotient of two functions: **bottom times derivative of top, minus top times derivative of bottom, all over bottom squared**