COLLEGE OF ENGINEERING AND APPLIED SCIENCES

NCSC 201: DISCRETE MATHEMATICS

END OF FIRST SEMESTER EXAMINATIONS

NOVEMBER 2024

LECTURER: MR TIMOTHY MAKAMBWA

TIME: 3 HOURS

INSTRUCTIONS

You are required to answer questions as instructed in each section

Start **each** question on a new page in your answer booklet

Answer all questions in Section **A** and any three from Section **B**

Credit will be awarded for logical, systematic and neat presentations

Section A :(40 marks)

Answer all questions in this Section

Question One

Each student in Liberal Arts at some college has a mathematics requirement *A* and a science requirement *B*. A poll of 140 sophomore students shows that: 60 completed *A*, 45 completed *B*, 20 completed both *A* and *B*.

Use a Venn diagram to find the number of students who have completed:

a. At least one of *A* and *B*; [3]

b. exactly one of *A* or *B*; [3]

c. neither *A* nor *B*. [2]

Question Two

Suppose $a = 5880$ and $b = 8316$.

a. Express *a* and *b* as products of primes. [4]

b. Find gcd (a, b) and lcm(a, b). [4]

c. Verify that lcm$(a, b) = |ab|/$ gcd(a, b). [2]

Question Three

a. Find the 10^{th} term of the A. P.: 2, 4, 6, ... [2]

b. The 10^{th} term of an A. P. is – 15 and 31^{th} term is –57, find the 15^{th} term. [4]

c. Find the 6^{th} term of the G. P.: 4, 8, 16, ... [2]

d. The 4^{th} and the 9^{th} term of a G. P. are 8 and 256 respectively. Find the 7^{th} term of the G. P. [4]

Question Four

Prove that

a. The product of two even integers is divisible by 4, [3]

b. The product of an even integer and an odd integer is even. [3]

c. Prove that if *n* is an integer, then $n^3 - n$ is always divisible by 6 [4]

Section B (60 marks)

Question Five

a. The 35^{th} term of an A. P. is 69. Find the sum of its 69 terms. [5]

b. The first term of an A. P. is 10, the last term is 50. If the sum of all the terms is 480, find the common difference and the number of terms. [5]

c. Find the sum of the G. P.: 1, 3, 9, 27, ... up to the 10th term. [5]

d. Find the sum of the G. P.: $1/\sqrt{3}$;1, $\sqrt{3}$,…..81 [5]

Question Six

a. Consider the two integers 125 and 962.

   i. Write down the prime decomposition of each of the two numbers.

   ii. Find their greatest common divisor.

   iii. Find their least common multiple.

b. Factorize the number 6469693230.

c. Find (210; 858). Determine integers x and y such that (210; 858) = 210x + 858y. Hence give the general solution of the equation in integers x and y.

d. Find (182; 247). Determine integers x and y such that (182; 247) = 182x + 247y. Hence give the general solution of the equation in integers x and y.

[20]

Question Seven

a. In a survey of university students, 64 had taken mathematics course, 94 had taken chemistry course, 58 had taken physics course, 28 had taken mathematics and physics, 26 had taken mathematics and chemistry, 22 had taken chemistry and physics course, and 14 had taken all the three courses. Find how many had taken one course only. [6]

b. In a group of students, 65 play football, 45 play hockey, 42 play cricket, 20 play football and hockey, 25 play football and cricket, 15 play hockey and cricket and 8 play all the three games. Find the total number of students in the group (Assume that each student in the group plays at least one game.) [6]

c. In a town 85% of the people speak Tamil, 40% speak English and 20% speak Hindi. Also 32% speak Tamil and English, 13% speak Tamil and Hindi and 10% speak English and Hindi, find the percentage of people who can speak all the three languages. [8]

Question Eight

a. Solve the recurrence relation $a_n = 7a_{n-1} - 10a_{n-2}$ with $a_0=2$ and $a_1= 3$. [6]

b. Solve the recurrence relation $a_n = 6a_{n-1} - 9a_{n-2}$ with initial conditions $a_0=1$ and $a_1=4$.[6]

c. Solve the recurrence relation $a_n= 2a_{n-1} - a_{n-2}$

i. What is the solution if the initial terms are $a_0= 1$ and $a_1=2$?

ii. What do the initial terms need to be in order for $a_9=30$?

iii. For which x are there initial terms which make $a_9=x$? [8]

Question Eight

a. Prove that $7^n - 1$ is a multiple of 6 for all $n \in \mathbb{N}$. [5]

b. Prove that $1+3+5+\cdots+(2n-1) = n^2$, for all $n \geq 1$. [5]

c. Prove that $F_0+F_2+F_4+\cdots+F_{2n}=F_{2n+1} - 1$, where F_n is the nth Fibonacci number. [5]

d. Prove that $2^n < n!$ For all $n \geq 4$ (Recall, $n! = 1\cdot 2\cdot 3\cdot\cdots\cdot n$.) [5]

END OF PAPER