“Investing in Africa’s Future”

COLLEGE OF ENGINEERING AND APPLIED SCIENCES(CEAS)

NMMS 105: MATHEMATICS FOR BUSINESS 2

END OF FIRST SEMESTER EXAMINATIONS

NOVEMBER 2024

LECTURER: Mr. Timothy Makambwa

DURATION: 3 HOURS

INSTRUCTIONS

Answer **ALL** the questions in Section A and **any** Three questions from Section B and each question has **20** marks. Total possible mark is **100.**

Start **each** question on a new page on your answer sheet.

The marks allocated to **each** question are shown at the end of the section.

Section A (40 Marks)

Answer ***all*** questions in this Section

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Question One

Evaluate

a) $\frac{d}{dx}(5x^{6})$ [3]

b) $\frac{d}{dx}(2x^{1.5})$ [3]

c) $\frac{d}{dx}(6x^{4}-x^{3})$ [4]

d) $\frac{d}{dx}(3x^{2}+5x+1)$ [5]

e) $\frac{d}{dx}(4x^{0.5}-2x^{-2}-7)$ [5]

Question Two

Integrate the following:

a) $\int 30x^{4}\,dx$ [3]

b) $\int (24+7.2x^{-2})\,dx$ [3]

c) $\int ((0.5/x)+1)\,dx$ [4]

d) $\int (48x-0.4e^{-1.4x})\,dx$ [5]

e) $\int (65+3^{x})\,dx$ [5]

Section B (60 Marks)

Answer any *three* questions from this Section B

Question Three

a. A monopoly faces the following TR and TC schedules:

$TR = 300q - 2q^2$

$TC = 12q^3 - 44q^2 + 60q + 30$

What output should it sell to maximize profit? [10]

b. A firm faces the demand function $p = 190 - 0.6q$ and the total cost function $TC = 40 + 30q + 0.4q^2$

(i) What output will maximize profit?

(ii) What output will maximize total revenue?

(iii) What will the output be if the firm makes a profit of £4,760? [10]

Question Four

a) Given the demand function $q = (1{,}200 - 2p)^{0.5}$, what is elasticity of demand when quantity is 30? [10]
b) What is elasticity of demand when quantity is 8 if a firm's demand function is $q = 60-2p^{0.5}$ (Where $p^{0.5} \geq 0$, $q \leq 60$)? [10]

Question Five

a) Find the area under the curve of the function y = $y = x^2 - 4x + 5$ from x = -1 to x =3. Sketch the graph [5]
b) Investigate the maxima and minima of the function $2x^3 + 3x^2 - 36x + 10$. [7]
c) Find the absolute (global) maximum and minimum values of the function $f(x) = 3x^5 - 25x^3 + 60x + 1$ in the interval [-2, 1] [8]

Find the area under the curve $y = x^2 - 4x + 5$ from x = -1 to x=3. Sketch the region.

Question Six

a) A Transport Company has two types of trucks, Type A and Type B. Type A has a refrigerated capacity of 20 m^3 and a non-refrigerated capacity of 40 m^3 while Type B has the same overall volume with equal sections for refrigerated and non-refrigerated stock. A grocer needs to hire trucks for the transport of 3,000 m^3 of refrigerated stock and 4 000 m^3 of non-refrigerated stock. The cost per kilometre of a Type A is $30, and $40 for Type B. How many trucks of each type should the grocer rent to achieve the minimum total cost? [10]
b) A School is preparing a trip for 400 students. The company who is providing the transportation has 10 buses of 50 seats each and 8 buses of 40 seats, but only has 9 drivers available. The rental cost for a large bus is $800 and $600 for the small bus. Calculate how many buses of each type should be used for the trip for the least possible cost. [10]

Question Seven

a) Solve by matrix method the equations $2x + 3y = 7$, $2x + y = 5$. [5]

b) Solve by matrix method the equations [15]

$x - 2y + 3z = 1$, $3x - y + 4z = 3$, $2x + y - 2z = -1$

Question Eight

a) For the marginal cost function $MC = 5 - 6x + 3x^2$, x is the output. If the cost of producing 10 items is Rs.850, find the total cost and average cost function. [5]

b) The marginal cost function is $MC = 20 - 0.04x + 0.003x^2$ where x is the number of units produced. The fixed cost of production is Rs. 7,000. Find the total cost and the average cost. [5]

c) If the marginal revenue function is R¢ (x) = $15 - 9x - 3x^2$, find the revenue function and average revenue function. [5]

d) If the marginal revenue of a commodity is given by $MR = 9 - 2x + 4x^2$, find the demand function and revenue function. [5]

Question Nine

a) The demand and supply function for a commodity are pd = 16- 2x and ps = x2 + 1. Find the consumers' surplus and producers' surplus at the market equilibrium price. [10]

b) The demand and supply law under a pure competition are given by $p_d = 23 – x^2$ and $p_s = 2x^2 - 4$. Find the consumers' surplus and producers' surplus at the market equilibrium price. [10]

END OF EXAMINATION

Mathematical   Formulae

Integration Techniques

Many integration formulas can be derived directly from their corresponding derivative formulas, while other integration problems require more work. Some that require more work are substitution and change of variables, integration by parts, trigonometric integrals, and trigonometric substitutions.

Basic formulas

Most of the following basic formulas directly follow the differentiation rules.

1. $\int kf(x)\,dx = k\int f(x)\,dx$
2. $\int [f(x) \pm g(x)]\,dx = \int f(x)\,dx \pm \int g(x)\,dx$
3. $\int k\,dx = kx + C$
4. $\int x^n\,dx = \frac{x^{n+1}}{n+1} + C, n \neq -1$
5. $\int \sin x\,dx = -\cos x + C$
6. $\int \cos x\,dx = \sin x + C$
7. $\int \sec^2 x\,dx = \tan x + C$
8. $\int \csc^2 x\,dx = -\cot x + C$
9. $\int \sec x \tan x\,dx = \sec x + C$
10. $\int \csc x \cot x\,dx = -\csc x + C$
11. $\int e^x\,dx = e^x + C$
12. $\int a^x\,dx = \frac{a^x}{\ln a} + C, a > 0, a \neq 1$
13. $\int \frac{dx}{x} = \ln|x| + C$
14. $\int \tan x\,dx = -\ln|\cos x| + C$
15. $\int \cot x\,dx = \ln|\sin x| + C$

16. $\int \sec x\,dx = \ln|\sec x + \tan x| + C$

17. $\int \csc x\,dx = -\ln|\csc x + \cot x| + C$

18. $\int \frac{dx}{\sqrt{a^2 - x^2}} = \arcsin\frac{x}{a} + C$

19. $\int \frac{dx}{a^2 + x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$

20. $\int \frac{dx}{x\sqrt{x^2 - a^2}} = \frac{1}{a}\operatorname{arcsec}\frac{x}{a} + C$

Applications of Integration

∴ Producer Surplus = p_0q_0 - ∫gxdx with limits from 0 to q_0

Consumer Surplus = ∫gxdx – p_0q_0 with limits from 0 to q_0

Total Cost =∫ MCdx

Total Revenue =∫MRdx

Differentiation

The chain rule

If *y* is a function of *u*, which is itself a function of *x*, then

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$$

differentiate the outer function and multiply by the derivative of the inner function

The product rule

If $y = uv$ then = $\frac{dy}{dx} = u.\frac{dv}{dx}$ x $v.\frac{du}{dx}$

This rule tells you how to differentiate the product of two functions: **multiply each function by the derivative of the other and add**

The quotient rule

If $y = \frac{u}{v}$ then $\frac{dy}{dx} = \frac{v.du/dx - u.dv/dx}{v^2}$

This rule tells you how to differentiate the quotient of two functions: **bottom times derivative of top, minus top times derivative of bottom, all over bottom squared**