COLLEGE OF BUSINESS, PEACE, LEADERSHIP AND GOVERNANCE

NMMS105: MATHEMATICS FOR BUSINESS 2

END OF SECOND SEMESTER EXAMINATIONS

MAY 2021

LECTURER: TARAMBAWAMWE P

DURATION: 7 HOURS

INSTRUCTIONS

You are required to answer only ONE question.

Credit will be awarded for logical, systematic and neat presentations

Q1 (a) i. Given B = $\begin{pmatrix} 2 & X-2 & 1 \\ 1 & 2 & -1 \\ 3 & 4-2x & -2 \end{pmatrix}$ Find X if det(B) = -6 [5]

ii. Let A =

$$\begin{bmatrix} -8 & 4 & -3 & 2 \\ 2 & 1 & -1 & 0 \\ -3 & -5 & 4 & 0 \\ 2 & -4 & 3 & -1 \end{bmatrix}.$$

Find the inverse of A using row operations [10]

(b) Solve the system of equations [8]

$-x_1 + 5x_2 = -8$

$-2x_1 + 5x_2 + 5x_3 + 2x_4 = 9$

$-3x_1 - x_2 + 3x_3 + x_4 = 3$

$7x_1 + 6x_2 + 5x_3 + x_4 = 30$

c.

i. Eva had a bake sale to earn extra money. On the first day, she earned $12.50 selling 10 cookies and 4 brownies. On the second day, she earned $15.50 selling 6 brownies and 8 pieces of pie. On the third day, she earned $12.00 selling 16 cookies. If Eva sold 12 cookies and 2 pieces of pie the next day, how much did she make? [5]

ii. John inherited $25,000 and invested part of it in a money market account, part in municipal bonds, and part in a mutual fund. After one year, he received a total of $1,620 in simple interest from the three investments. The money market paid 6% annually, the bonds paid 7% annually, and the mutually fund paid 8% annually. There was $6,000 more invested in the bonds than the mutual funds. Find the amount John invested in each category [8]

III. . A person is repaying a loan of $50000 at $200 per month. The interest rate is 3% per month. Form a difference equation, solve the difference equation and find how long it will take to repay the loan. [7]

IV. e. . A cup of coffee is initially 170 degrees Fahrenheit and is left in a room with ambient temperature 70 degrees Fahrenheit. Suppose that when the coffee is first placed in the room, it is cooling at a rate of 20 degrees per minute. Assuming Newton's law of cooling applies, how long does it take for the coffee to cool to 110 degrees? [7]

Q 2

a.

Find i. dy/dx for the $x^{-3} + y^3 = -6xy$ and at then ii. Find the equation of the normal to the curve $x^{-3} + y^3 = -6xy$ at the point (3;3) [5]

ii. Determine the following. $\int\left(x - 2x^2 + \frac{1}{3x}\right)dx$ [5]

b) find dy/dx for each of the following

i. $y = 4e^{-x}(1 - \ln x)$ [5]

ii. $y = e^{-2x}\ln(5x^{-3} + x^2)$ [5]

c. Eva had a bake sale to earn extra money. On the first day, she earned $12.50 selling 10 cookies and 4 brownies. On the second day, she earned $15.50 selling 6 brownies and 8 pieces of pie. On the third day, she earned $12.00 selling 16 cookies. If Eva sold 12 cookies and 2 pieces of pie the next day, how much did she make? [10]

d. A person is repaying a loan of $50000 at $200 per month. The interest rate is 3% per month. Form a difference equation, solve the difference equation and find how long it will take to repay the loan. [10]

e. . A cup of coffee is initially 170 degrees Fahrenheit and is left in a room with ambient temperature 70 degrees Fahrenheit. Suppose that when the coffee is first placed in the room, it is cooling at a rate of 20 degrees per minute. Assuming Newton's law of cooling applies, how long does it take for the coffee to cool to 110 degrees? [10]

Q3

a) Solve the (separable) differential equations

i. $\frac{dy}{dx} = \frac{x^2 y - 4y}{x + 2}$ [4]

ii. $\frac{dy}{dx} = x e^{x^2 - \ln(y^2)}$ [4]

b)

i. A cup of coffee is initially 170 degrees Fahrenheit and is left in a room with ambient temperature 70 degrees Fahrenheit. Suppose that when the coffee is first placed in the room, it is cooling at a rate of 20 degrees per minute. Assuming Newton's law of cooling applies, how long does it take for the coffee to cool to 110 degrees? [7]

ii. The population of fish in a pond is modelled by the differential equation

$$\frac{dP}{dt} = 480 - 4P$$

where time t is measured in years. Towards what number does the population of fish tend? If there are initially 10 fish in the pond, how long does it take for the number of fish to reach 90% of the eventual population? [7]

c) Find the general and particular solutions of the following and determine the general path of each solution

i. $\mathbf{Y_{t+1}} = 10Y_t - 20$, given $Y_0 = 20$ [5]

ii. $\mathbf{P_{t+1}} = 0.85P_t + 9(3)^t$, given $P_0 = 2000$ [7]

d) The population of Zimbabwe is 15 million. The birth rate is 45 per year and the death rate is 3% per year. There are 50000 people who get into Zimbabwe every year from other countries.

i. Form a difference equation [3]
ii. Solve the difference equation and then estimate the population in 20 years. [5]

e) A person is repaying a loan of $50000 at $200 per month. The interest rate is 3% per month

i. Form a difference equation. [3]
ii. Solve the difference equation and find how long it will take to repay the loan. [5]

END OF EXAMINATION