AFRICA UNIVERSITY
A United Methodist-Related Institution

"Investing in Africa's Future"

COLLEGE OF ENGINEERING AND APPLIED SCIENCES (CEAS)

NMMS 105: MATHEMATICS FOR BUSINESS 2

END OF SECOND SEMESTER EXAMINATIONS

APRIL 2025

LECTURER: MR E. CHIKAKA

TIME: 3 HOURS

INSTRUCTIONS

You are required to answer questions as instructed in each section

Start each question on a new page in your answer booklet

Answer all questions in Section A and any three from Section B

Section A: Compulsory (40 Marks)

1. a) Differentiate the following functions with respect to x:

   i) $f(x)=5x^3-3x^2+2x-7$ [3]

   ii) $g(x)=e^{2x}\sin(x)$ [4]

2. a) Evaluate the following integrals:

   i) $\int(4x^3-2x)\,dx$ [3]

   ii) $\int e^x\cos(x)\,dx$ [4]

3. A company produces two products, A and B. Each unit of A requires 2 hours of labour and 3 units of material. Each unit of B requires 4 hours of labour and 2 units of material. The company has a maximum of 100 hours of labour and 90 units of material available.

   Maximize and minimize $P=5x+7y$ [10]

4. a) Given the matrices $A=\begin{pmatrix} 2 & 3 \\ 1 & 4 \end{pmatrix}$ and $B=\begin{pmatrix} 5 & 1 \\ 2 & 3 \end{pmatrix}$, calculate:

   i) A+B ii) A^T [4]

   iii) $(AB)^{-1}$ [4]

5. a) Solve the differential equation $\frac{dy}{dx} + 3y = 6$, given that y (0)=2 [4]

6. a) Solve the difference equation $y_{n+1} = 0.8y_n + 5$ with $y_0=10$ [4]

Section B: Answer ANY 3 Questions (Each Question carries 20 Marks)

Question 1

a) Find the second derivative of $f(x)=x^4-4x^3+ 6x- 4x + 1$ [5]

b) Determine the local maxima and minima of f(x) [10]

c) A company's cost function is $C(x)=50x+2000C(x) = 50x + 2000$. The revenue function is $R(x)=80x$. Find the production level that maximizes profit. [5]

Question 2

a) Evaluate $\int(x^2+3x+2)\,dx$

b) Find the area under the curve $y=x^3-x$ from x=0 to x=2

c) A company's marginal cost function is $MC(x)=10+0.5x$. Find the total cost function given that the fixed cost is $500.

Question 3

A factory produces two products, P and Q. The profit per unit of P is \$4 and for Q is \$6. Each unit of P requires 2 hours of machine time and 1 unit of raw material. Each unit of Q requires 3 hours of machine time and 2 units of raw material. The factory has 60 hours of machine time and 40 units of raw material available.

a) Formulate the linear programming model to maximize profit. [6]

b) Solve graphically and determine the optimal production levels. [10]

c) Calculate the maximum profit. [4]

Question 4

a) Find the inverse of the matrix $A = \begin{pmatrix} 2 & 1 & 1 \\ 6 & 5 & -3 \\ 4 & -1 & 3 \end{pmatrix}$ [5]

b) Hence or otherwise solve the equation

$$\begin{aligned} 2x+y+z &= 12 \\ 6x+5y-3z &= 6 \\ 4x-y+3z &= 5 \end{aligned}$$ **[10]**

c) Solve using matrices the equations $2x-y = 3$, $5x+y = 4$. **[5]**

Question 5

a) Solve the differential equation $\frac{dy}{dx} = 2x - y$ [8]

b) A business's revenue grows at a rate proportional to its current revenue. Write and solve the differential equation given that the revenue is \$1000 when t=0 and grows at 5% per year. [12]

Question 6

a) Solve the difference equation $y_{n+1} - y_n = 3$ [8]

b) A company's profit changes according to the equation $P_{n+1} = 0.9 P_n + 100$, where $P_0 = 500$. Find P_1, P_2, and the general solution. [12]

END OF EXAMINATION