"Investing in Africa's Future"

COLLEGE OF ENGINEERING AND APPLIED SCIENCES (CEAS)

CSC 1103: CALCULUS

END OF FIRST SEMESTER EXAMINATIONS

NOVEMBER 2025

LECTURER: MR TIMOTHY MAKAMBWA

TIME: 3 HOURS

INSTRUCTIONS

You are required to answer questions as instructed in each section

Start each question on a new page in your answer booklet

Answer all questions in Section A and any three from Section B

Section A – (Compulsory 40 Marks)

Answer all questions in this Section

Question One

Find the limits of the following functions

a) $\lim_{x\to-\infty}\frac{3x^7-7x^2+1}{2x^4+x-3}$ [2]

b) $\lim_{x\to\infty}\frac{3x^4-x^3+2}{2x^4+x-3}$ [2]

c) $\lim_{x\to-\infty}\frac{3x^1+2}{x^3+x-3}$ [3]

d) $\lim_{x\to1}\frac{\sqrt{x+3}-2}{x-1}$ [3]

e) $\lim_{x\to1}\frac{4x^2+x-5}{x-1}$ [3]

f) $\lim_{x\to0}\frac{x+Sinx}{x}$ [3]

g) $\lim_{x\to0}\frac{1-Cos(2x)}{4x}$ [3]

Question Two

Differentiate the following functions

a) $f(x) = 4x^5 \ -5x^4$ [2]

b) $f(x) = e^x \sin x$ [3]

c) $f(x)=\frac{x}{1+x\ \hat{}2}$ [3]

d. Find $\frac{d^3y}{dx^3}$ for $y = 2x^5 + 3x^3 + 9x - 5e^x$ [3]

Question Four

Integrate the following functions

a) $\int (2x - \frac{1}{x})$dx [2]

b) $\int_1^e (x^2 - \frac{1}{x})dx$ [3]

c) $\int \left(\frac{1-tanx}{1\mp tanx}\right) dx$ [4]

d) find $f(x)$ if $f'(x) = \sqrt{x} - 3$ and $f(4) = -1.$. [4]

Section B (60 Marks)

Answer Any Three questions

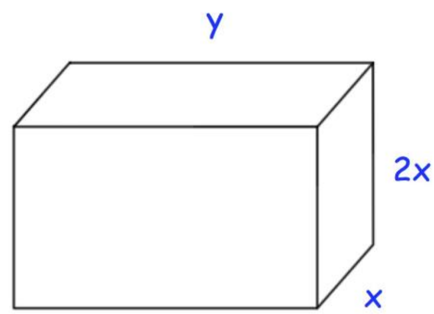
Question Five

a) A firm faces the demand schedule $q = 200 - 4p$ and the cost schedule TC $= 0.1q^3 - 0.5q^2 + 2q + 8$. What price will maximize profit? [12]

b) Given the demand function $q = (1{,}200 - 2p)^{0.5}$, what is elasticity of demand when quantity is 30? [8]

Question Six

a) Shown below is a cuboid.



The surface area of the cuboid is 120cm².

i. Show that $y = \frac{20}{x} - \frac{2x}{3}$ [3]

ii. Show that the volume of the cuboid is given by
$V = 40x - \frac{4}{3}x^3$ [3]

iii. Use differentiation to find the value of x for which V is a maximum [4]

iv. Use your answer to (iii) to find the maximum volume of the cuboid. [3]

b) The volume of a container with a height of x, is given by
$V = x(x-1)(9-x)$ where $1 < x < 9$

i. Find $\frac{dV}{dx}$ [3]

ii. Hence or otherwise find the value of x for which the volume is a maximum. Give your answer to 1 decimal place. [4]

Question Seven

a) Find the equation of the tangent and normal to the demand curve y = 10-3x^2 at (1, 7). [5]
b) Find the slope of the tangent line at the point (0, 5) of the curve y =⅓ (x^2 + 10x ▯▯15). At what point of the curve the slope of the tangent line is 5 /8 ? [5]
c) Determine the coefficients a and b so that the curve y = ax^2 ▯6x +b may pass through the point (0, 2) and have its tangent parallel to the x-axis at x = 1.5. [5]
d) Determine the values of l and m so that the curve, y = lx^2+ 3x + m may pass through the point (0, 1) and have its tangent parallel to the x-axis at x = 0.75. [5]

Question Eight

a) If the demand function is p = 35 ▯▯2x ▯▯$x^2$and the demand x0 is 3, find the consumers' surplus. [6]

b) If the supply law is p = 4 ▯▯x + x^2, find the producers' surplus when the price is 6. [6]
c) The demand and supply law under a pure competition are given by pd= 23 ▯▯$x^2$and ps= 2x^2▯▯4. Find the consumers' surplus and producers' surplus at the market equilibrium price. [8]

Question Nine

a) For the marginal cost function MC = 5 - 6x + 3x^2, x is the output. If the cost of producing 10 items is Rs.850, find the total cost and average cost function. [5]

b) The marginal cost function is MC = 20 - 0.04x + 0.003x^2 where x is the number of units produced. The fixed cost of production is Rs. 7,000. Find the total cost and the average cost. [5]

c) If the marginal revenue function is R¢ (x) = 15 - 9x - 3x^2, find the revenue function and average revenue function. [5]

d) If the marginal revenue of a commodity is given by MR = 9 - 2x + 4x^2, find the demand function and revenue function. [5]

END OF EXAMINATION

Mathematical Formulae

Integration Techniques

Many integration formulas can be derived directly from their corresponding derivative formulas, while other integration problems require more work. Some that require more work are substitution and change of variables, integration by parts, trigonometric integrals, and trigonometric substitutions.

Basic formulas

Most of the following basic formulas directly follow the differentiation rules.

1. $\int kf(x)\,dx = k\int f(x)\,dx$
2. $\int [f(x) \pm g(x)]dx = \int f(x)\,dx \pm \int g(x)\,dx$
3. $\int k dx = kx + C$
4. $\int x^n\,dx = \frac{x^{n+1}}{n+1} + C, n \neq -1$
5. $\int \sin x\,dx = -\cos x + C$
6. $\int \cos x\,dx = \sin x + C$
7. $\int \sec^2 x\,dx = \tan x + C$
8. $\int \csc^2 x\,dx = -\cot x + C$
9. $\int \sec x \tan x\,dx = \sec x + C$
10. $\int \csc x \cot x\,dx = -\csc x + C$
11. $\int e^x\,dx = e^x + C$
12. $\int a^x\,dx = \frac{a^x}{\ln a} + C, a > 0, a \neq 1$
13. $\int \frac{dx}{x} = \ln|x| + C$
14. $\int \tan x\,dx = -\ln|\cos x| + C$
15. $\int \cot x\,dx = \ln|\sin x| + C$
16. $\int \sec x\,dx = \ln|\sec x + \tan x| + C$

17. $\int \csc x \, dx = -\ln|\csc x + \cot x| + C$

18. $\int \frac{dx}{\sqrt{a^2 - x^2}} = \arcsin\frac{x}{a} + C$

19. $\int \frac{dx}{a^2 + x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$

20. $\int \frac{dx}{x\sqrt{x^2 - a^2}} = \frac{1}{a}\text{arcsec}\frac{x}{a} + C$

Applications of Integration

∴ Producer Surplus = $p_0q_0 - \int gx\,dx$ with limits from 0 to q_0

Consumer Surplus = $\int gx\,dx - p_0q_0$ with limits from 0 to q_0

Total Cost = ∫ MCdx

Total Revenue =∫MRdx

Differentiation

The chain rule

If *y* is a function of *u*, which is itself a function of *x*, then

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$$

differentiate the outer function and multiply by the derivative of the inner function

The product rule:

If $y = uv$ then = $\frac{dy}{dx} = u.\frac{dv}{dx} \times v.\frac{du}{dx}$

This rule tells you how to differentiate the product of two functions: **multiply each function by the derivative of the other and add**

The quotient rule:

If $y = \frac{u}{v}$ then $\frac{dy}{dx} = \frac{v.du/dx - u.dv/dx}{v^2}$

This rule tells you how to differentiate the quotient of two functions: **bottom times derivative of top, minus top times derivative of bottom, all over bottom squared**