AFRICA UNIVERSITY
A United Methodist-Related Institution

"Investing in Africa's Future"

COLLEGE OF ENGINEERING AND APPLIED SCIENCES

NCSC 101: DISCRETE STRUCTURES

END OF FIRST SEMESTER EXAMINATIONS

NOVEMBER 2025

LECTURER: Mr. Timothy Makambwa

DURATION: 3 HOURS

INSTRUCTIONS

Answer **ALL** the questions in **Section A** and any Three questions from Section B and each question has **20** marks. Total possible mark is **100.**

Start **each** question on a new page on your answer sheet.

The marks allocated to **each** question are shown at the end of the section.

Section A – (Compulsory 40 Marks)

Question One

Using the truth table or otherwise, check that each of the following is tautology

a) $P \rightarrow (P \vee Q)$

b) $((P \wedge -Q) \rightarrow (P \rightarrow Q)$

c) $P \rightarrow (Q \rightarrow P)$

d) $(P \vee (P \wedge Q) \leftrightarrow P$

e) $(P \vee Q) \rightarrow (Q \rightarrow (P \wedge Q)$

[4x5]

Question Two

a)

Prove that $1 \cdot 1! + 2 \cdot 2! + \ldots + n \cdot n! = (n+1)! - 1, \quad \forall$ positive integers

[10]

b) Prove by induction that the following statements are true for all integers n $4007^n - 1$ is divisible by 2003 [10]

Section B (60 Marks)

Answer any *three* in this Section

Question Three

Prove the following formulas for all positive integers n.

a) $1 + 2 + 3 + 4 + 5 + \ldots + n = n(n + 1)/ 2$
b) $2 + 4 + 6 + 8 + 10 + \ldots + 2n = n^2 + n$
c) $1 + 2 + 4 + 8 + 16 + \ldots + 2^{n-1} = 2^n - 1$
d) $1 + 3 + 9 + 27 + 81 + \ldots + 3^{n-1} = (3^n - 1)/ 2$
e) $1 + 4 + 9 + 16 + 25 + \ldots + n^2 = n(n + 1)(2n + 1)/ 6$

[4x5]

Question Four

Prove by induction for all positive integers n.

a) $2^{2n} - 1$ is a multiple of 3
b) 7 is a divisor of $2^{3n} - 1$
c) $n^3 + 2n$ is a multiple of 3
d) $n^5 - n \bmod 5 = 0$
e) $2^{n+2} + 3^{2n+1}$ is a multiple of 7

[4x5]

Question Five

Prove the following propositions.

a) $n < 2^2 \ \forall n \geq 1$

b) $2^n < n! \ \forall n \geq 4$

c) $3^n < n! \ \forall n \geq 7$

d) $2^n > n^n \ \forall n \geq 5$

e) $n! < n^n \ \forall n \geq 2$

[4x5]

Question Six

Prove the following propositions using proof by contrapositive
a) If x^2 is even then x must be even.
b) If x^3 is even then x must be even.
c) If $x^2 - 2x$ is even then x must be even.
d) If $x^3 - 4x + 2$ is odd then x must be odd.

[4x5]

Question Seven

Part A

Suppose a = 5880 and b = 8316.

a) Express a and b as products of primes. [2]
b) Find gcd (a, b) and lcm(a, b). [4]
c) Verify that lcm(a, b) = |ab|/ gcd(a, b). [2]

Part B
Find the Greatest Common Divisors (GCD) of the following pairs

a) 275 and 115

b) 999 and 123

c) 456 and 144

d) 725 and 1000

e) 1111 and 11111

[2x5]

Question Eight

a) A = {1,2,3}
R = {(1,2),(2,1)}

R = {(1,1),(2,2),(3,3)}

R = {(1,1),(2,2),(3,3),(1,2)}

Find out whether the relations are symmetric, reflexive, or transitive. [10]

b) (a,b) ε R if a-b is a multiple of 3. Show that it is an equivalence relation i.e.it is reflexive, symmetric, and transitive. [10]

END OF EXAMINATION