AFRICA UNIVERSITY
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"Investing in Africa's future"

COLLEGE OF BUSINESS, PEACE, LEADERSHIP AND GOVERNANCE

NMEC203: MATHEMATICS FOR ECONOMICS

END OF FIRST SEMESTER EXAMINATIONS

NOVEMBER 2021

LECTURER: DR SAUNGWEME

TIME: 5 HOURS

INSTRUCTIONS

1) ANSWER ANY ONE QUESTION

2) ALL QUESTIONS CARRY EQUAL MARKS

QUESTION 1

(a) Suppose the monthly marginal revenue function is given by $R(x) = 10 - 0.01x + \frac{150}{x+2}$. Calculate the revenue function. [5 marks]

(b) Suppose that a firm operates with the total cost function $TC = 50 + 0.4q^2$ and is a monopoly facing the demand schedule $p = 360 - 2.1q$.

(i) Derive the firm's net profit function. [5 marks]

(ii) How many units should the firm sell to maximise profits? [5 marks]

(iii)Suppose the government charges a tax t on each unit sold:

a) What is the comparative static effect of the tax on the output in c(ii) above? [5 marks]

b) What is the effect of a \$1 increase in per-unit tax on the firm's profit maximising price? [5 marks]

(c) Find the first derivatives of the following functions:

(i) $y = 5x^2e^{-4x}$. [5 marks]

(ii) $y = ln\left(\frac{3x-1}{1-x}\right)$. [5 marks]

(d) Simplify the expression $\frac{3(2^{n+1})-4(2^{n-1})}{2^{n+1}-2^n}$. [5 marks]

(e) Solve the following equations:

(i) $5^{2x} - 5^{x+1} + 4 = 0$. [5 marks]

(ii) $x^2 + 2x + \frac{12}{x^2+2x} = 7$. [5 marks]

(f) Find the domain and range of the function $f(x) = \frac{2x-1}{x^2-x}$. [5 marks]

(g) Sketch the graph of the function $g(x) = 2 - (x + 2)^{-2}$. [5 marks]

[TOTAL 60 MARKS]

QUESTION 2

(a) The demand and supply functions for a good are given by:

$$P_d = 100 - 0.5Q_d$$
$$P_s = 10 + 0.5Q_s$$

Analyse the effect of the introduction of a price ceiling of \$40 in this market. [10 marks]

(b) Find the first derivatives of the following functions with respect to x:

(i) $f(x) = -5x^{-3} + 4x^3 + 2x + 7$ [2 marks]

(ii) $y = x^5e^{x^5}$ [3 marks]

(iii) $f(x) = \frac{\sqrt{x}-2}{\sqrt{x}+1}$ [5 marks]

(c) A firm's total cost function is given by $TC = 40 + 82q - 6q^2 + 0.2q^3$.

At what quantity will the average variable cost be at its minimum? [10 marks]

(d) Suppose the total profit made by Saungweme engineering firm is given by the equation:

$$p = -x^2 + 24x + 5000$$

where x is the number of clients the firm has and p is the profit. Find the maximum profit made by the company. [5 marks]

(e) A firm's demand schedule is given by:

$$p = 80 - 2q$$

(i) Derive expressions for the firm's total revenue and marginal revenue functions. [5 marks]

(ii) Draw the firm's demand, marginal and total revenue curves on the same cartesian plane. [5 marks]

(f) Find the possible stationary points of the following function $f(x) = x^2e^{-x}$ on [0,4]. [5 marks]

(g) Solve the following equations:

(i) $x^2 + 3x - 2 = \frac{8}{x^2+3x}$. [5 marks]

(ii) $\sqrt{5}x^2 + x + \sqrt{5} = 0$. [5 marks]

[TOTAL 60 MARKS]

QUESTION 3

(a) The demand schedule is given by P = 60 – 0.2Q. Calculate the price elasticity of demand if the price is \$24. [5 marks]

(b) Find the integer roots of the following equations:

(i) $x^4 - x^3 - 7x^2 + x + 6 = 0$. [5 marks]

(ii) $\frac{1}{4}x^4 + \frac{1}{4}x^2 - x + 1 = 0$. [5 marks]

(c) Find the domain and range of the functions below:

(i) $F(x) = \sqrt{x-5}$. [5 marks]

(ii) $F(x) = \frac{x^2-4}{x^2-8x+12}$. [5 marks]

(d) Simplify the following:

(i) $\frac{x-y}{x+y} - \frac{x}{x-y} + \frac{3xy}{x^2-y^2}$. [5 marks]

(ii) $\frac{2a^2-18b^2}{a^2+6ab+9b}$. [5 marks]

(e) If $x^{-4}y^6 = 5$, solve:

(i) $x^{-8}y^{12}$. [2 marks]

(ii) $x^{-4}y^6 + 2x^{12}y^{-18}$. [3 marks]

(f) Evaluate the following:

(i) $\int(3x^4 + 5x^2 + 2)dx$. [2 marks]

(ii) $\int_2^b e^{2x}dx$. [3 marks]

(g) Find the maximum and minimum values for:

(i) $f(x) = 3x^2 - 6x + 5 \qquad x \in [0,3]$. [5 marks]

(ii) $f(x) = \frac{1}{4}x^4 - \frac{5}{6}x^3 + \frac{1}{2}x^2 - 1 \qquad x \in [-1,3]$. [5 marks]

(h) Find the profit-maximising output for a firm with the total cost function $TC = 4 + 97q - 8.5q^2 + \frac{1}{3}q^3$ and the total revenue function $TR = 58q - 0.5q^2$. [5 marks]

[TOTAL 60 MARKS]

END OF EXAMINATION