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COLLEGE OF BUSINESS, PEACE, LEADERSHIP AND GOVERNANCE

MATHEMATICS FOR BUSINESS 2 –NMMS 105

END OF SECOND SEMESTER EXAMINATIONS

NOVEMBER 2021

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DURATION: 5 HOURS

INSTRUCTIONS

Answer ***One*** Question from this examination.

Start **each** question on a new page on your answer sheet.

The marks allocated to **each** Question are shown at the end of the section.

Question One

A1.

A firm produces x tonnes of output at a total cost $C(x) = 0.1\,x^3 - 4x^2 + 20x + 5$ Find

(i) Average cost
(ii) Average Variable Cost
(iii) Average Fixed Cost
(iv) Marginal Cost
(v) Marginal Average Cost [20]

A2.

The demand curve for a monopolist is given by $x = 100-4p$

(i) Find the total revenue, average revenue and marginal revenue.
(ii) At what value of x, the marginal revenue is equal to zero?

[10]

A3.

a) On a chicken farm, the poultry is given a healthy diet to gain weight. The chickens have to consume a minimum of 15 units of Substance A and another 15 units of Substance B. In the market there are only two classes of compounds: Type X, with a composition of one unit of A to five units of B, and another type, Y, with a composition of five units of A to one of B. The price of Type X is $10 and Type Y, $30. What are the quantities of each type of compound that have to be purchased to cover the needs of the diet with a minimal cost? [15]
b) The marginal cost function of manufacturing *x* units of a commodity is $6 + 10x - 6x^2$. Find the total cost and average cost, given that the total cost of producing 1 unit is 15. [10]
c) If the marginal revenue for a commodity is $MR = 9 - 6x^2 + 2x$, find the total revenue and demand function. [10]
d) The marginal cost function of manufacturing *x* units of a commodity is $3 - 2x - x^2$. If the fixed cost is 200, find the total cost and average cost functions. [10]

A4.

a) Solve the equations $x + 2y + 5z = 23$, $3x + y + 4z = 26$, $6x + y + 7z = 47$ by determinant method or otherwise. [15]
b) The demand and supply law under a pure competition are given by $pd = 23 - x^2$ and $ps = 2x^2 - 4$. Find the consumers' surplus and producers' surplus at the market equilibrium price. [10]

Question Two

B1.

a) Given the demand schedule p = 120 − 3q derive a function for MR and find the output at which TR is a maximum.

b) For the demand schedule p = 40 − 0.5q find the value of MR when q = 15.

c) Find the output at which MR is zero when $p = 720 - 4q^{0.5}$ describes the demand schedule.

d) A firm knows that the demand function for its output is p = 400 − 0.5q. What price should it charge to maximize sales revenue?

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[5+5+5+5]

B2.

a) A monopoly faces the demand schedule $p = 460 - 2q$ and the cost schedule $TC = 20 + 0.5q^2$
How much should it sell to maximize profit and what will this maximum profit be? (All costs and prices are in £.)

b) A firm faces the demand function $p = 190 - 0.6q$ and the total cost function $TC = 40 + 30q + 0.4q^2$
(i) What output will maximize profit?
(ii) What output will maximize total revenue?
(iii) What will the output be if the firm makes a profit of £4,760?

[8+12]

B3.

a) With the start of school approaching, a store is planning on having a sale on school materials. They have 600 notebooks, 500 folders and 400 pens in stock, and they plan on packing it in two different forms. In the first package, there will be 2 notebooks, 1 folder and 2 pens, and in the second one, 3 notebooks, 1 folder and 1 pen. The price of each package will be $6.50 and $7.00 respectively. How many packages should they
put together of each type to obtain the maximum benefit? [20]

b) For each of the differential equations below (i) derive the definite solution, and (ii) use this solution to predict the value of *y* when *t* = 10.

c)

d) i) $\frac{dy}{dt} = 0.2y$ with initial value $y_0 = 200$

ii) $\frac{dy}{dt} = 1.2y$ with initial value $y_0 = 45$

iii) $\frac{dy}{dt} = -0.4y$ with initial value $y_0 = 14$

iv) $\frac{dy}{dt} = 0.354y$ with initial value $y_0 = 40$

[20]

B4.

Solve each of the following difference equations for the indicated variable by the iteration method.

(a) $Y_{t+1} - 0.8 Y_t = 10$, given $Y_1 = 5$. Find Y_5.

(b) $P_{t+2} = 4P_{t+1} - 8P_t$ given $P_1 = 20$, $P_2 = 18$. Find P_5.

(c) $P_t = 0.6P_{t-1} + 80$, given $P_1 = 100$. Find P_5

[6+7+7]

Question Three

C1.

A firm faces the demand schedule $p = 200 - 2q$ and the total cost function $TC = \frac{2}{3}q^3 - 14q^2 + 222q + 50$

Derive expressions for the following functions and find out whether they have maximum or minimum points. If they do, say what value of q this occurs at and calculate the actual value of the function at this output.

(a) Marginal cost
(b) Average variable cost
(c) Average fixed cost
(d) Total revenue
(e) Marginal revenue
(f) Profit [20]

C2.

a) Given the demand function $q = (1{,}200 - 2p)^{0.5}$, what is elasticity of demand when quantity is 30? [10]

b) Investigate the maxima and minima of the function $2x^3 + 3x^2 - 36x + 10$. [10]

c) If the marginal revenue for a commodity is $MR = 9 - 6x^2 + 2x$, find the total revenue and demand function. [10]

C3.

a) Solve using matrices the equations $2x - y = 3$, $5x + y = 4$. [5]

b) Solve the equations $2x + 8y + 5z = 5$, $x + y + z = -2$, $x + 2y - z = 2$ by using matrix method or otherwise [15].

c) The demand and supply functions under pure competition are $p_d = 16 - x^2$ and $p_s = 2x^2 + 4$. Find the consumers' surplus and producers' surplus at the market equilibrium price. [15]

d) A School is preparing a trip for 400 students. The company who is providing the transportation has 10 buses of 50 seats each and 8 buses of 40 seats, but only has 9 drivers available. The rental cost for a large bus is \$800 and \$600 for the small bus. Calculate how many buses of each type should be used for the trip for the least possible cost. [15]

END OF EXAM

Mathematical Formulae

Integration Techniques

Many integration formulas can be derived directly from their corresponding derivative formulas, while other integration problems require more work. Some that require more work are substitution and change of variables, integration by parts, trigonometric integrals, and trigonometric substitutions.

Basic formulas

Most of the following basic formulas directly follow the differentiation rules.

20. $\int \frac{dx}{x\sqrt{x^2 - a^2}} = \frac{1}{a}\operatorname{arcsec}\frac{x}{a} + C$

Applications of Integration

□□Producer Surplus = p_0q_0 - □gx*dx* with limits from 0 to q_0

Consumer Surplus = □gx*dx* – p_0q_0 with limits from 0 to q_0

Total Cost =∫ MCdx

Total Revenue =∫MRdx

1. $\int kf(x)\,dx = k\int f(x)\,dx$
2. $\int [f(x) \pm g(x)]\,dx = \int f(x)\,dx \pm \int g(x)\,dx$
3. $\int k\,dx = kx + C$
4. $\int x^n\,dx = \frac{x^{n+1}}{n+1} + C, n \neq -1$
5. $\int \sin x\,dx = -\cos x + C$
6. $\int \cos x\,dx = \sin x + C$
7. $\int \sec^2 x\,dx = \tan x + C$
8. $\int \csc^2 x\,dx = -\cot x + C$
9. $\int \sec x \tan x\,dx = \sec x + C$
10. $\int \csc x \cot x\,dx = -\csc x + C$
11. $\int e^x\,dx = e^x + C$
12. $\int a^x\,dx = \frac{a^x}{\ln a} + C, a > 0, a \neq 1$
13. $\int \frac{dx}{x} = \ln|x| + C$
14. $\int \tan x\,dx = -\ln|\cos x| + C$
15. $\int \cot x\,dx = \ln|\sin x| + C$
16. $\int \sec x\,dx = \ln|\sec x + \tan x| + C$
17. $\int \csc x\,dx = -\ln|\csc x + \cot x| + C$
18. $\int \frac{dx}{\sqrt{a^2 - x^2}} = \arcsin\frac{x}{a} + C$
19. $\int \frac{dx}{a^2 + x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$

Differentiation

The chain rule

If *y* is a function of *u*, which is itself a function of *x*, then

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$$

differentiate the outer function and multiply by the derivative of the inner function

The product rule

If *y* = *uv* then $\frac{dy}{dx} = u.\frac{dv}{dx}$ x $v.\frac{du}{dx}$

This rule tells you how to differentiate the product of two functions: **multiply each function by the derivative of the other and add**

The quotient rule

If *y* = $\frac{u}{v}$ then $\frac{dy}{dx} = \frac{v.du/dx - u.dv/dx}{v^2}$

This rule tells you how to differentiate the quotient of two functions: **bottom times derivative of top, minus top times derivative of bottom, all over bottom squared**